Finite-time ruin probability for Markovian skip-free risk processes

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In this talk, we study the ruin problem for a risk process whose dynamics is given as an upward (irreducible) skip-free Markov chains. It means that the surplus process has upward jumps of unit size, yet it can have downward transition of arbitrary magnitude. More specifically, we provide, in terms of some fundamental excessive functions, an explicit representation for the moment generating function of the ruin time.

We mention that, in their seminal paper [4], Karlin and McGregor have shown that the first passage time upward from state x to state y, where $x < y$, is a convolution of geometric random variables for birth-and-death processes, that is for Markov chains which are both upward and downward skip-free. This fascinating result has been extended to upward skip-free Markov chains by two different interesting methods, namely a spectral approach by Abate and Whitt [1] and an intertwining approach by Fill [3] and Diaconis and Fill [2].

Unfortunately, little is known about the more delicate case when $x > y$, that is for the ruin time (i.e. y=0) which may occur by a jump as the skip-free property no longer apply. To circumvent this difficulty, we suggest an original and comprehensive approach based on a combination of potential theory and the theory of Martin boundary. The motivation underlying our approach is to take advantage of the upward skip-free property to characterize explicitly the upper Martin boundary, and, by means of tools from potential theory, we develop methodologies to study the first passage time downward. Finally, we shall explain how our approach enables us to get detailed information, regarding the (finite-time) ruin probability as well as the so-called Gerber-Shiu function in the framework of general discrete-time risk models. If time allows, we shall also discuss the delicate issue regarding the characterization of the spectrum of the chain transition matrix. In particular, we shall provide a set of (easy to check) sufficient conditions for the reality and simplicity of its spectrum.

References

[1] J. Abate and W. Whitt, *Spectral Theory for Skip-Free Markov Chains*, Probab. Engr. Inform. Sci, 3: 77-88 (1989).

[2] P. Diaconis and J.A. Fill, *Strong stationary times via a new form of duality*, Ann. Probab., 18: 1483?1522 (1990).

[3] J.A. Fill, *On Hitting Times and Fastest Strong Stationary Times for Skip-Free and More General Chains*, J. Theoret. Probab., 22: 587-600 (2009).

[4] S. Karlin and J. McGregor, *Coincidence properties of birth and death processes*, Pacific J. Math., 9, 1109-1140 (1959).