## Optimal dividend problem for two companies with a collaboration agreement

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## Abstract:

We consider a two-dimensional optimal dividend problem in the context of two insurance companies with independent compound Poisson surplus processes, which collaborate by paying each other's deficit when possible. We solve the stochastic control problem of maximizing the weighted sum of expected discounted dividend payments (among all admissible dividend strategies) until ruin of both companies, by extending results of univariate optimal control theory. We see that the collaboration procedure is advantageous over merging the two companies completely when the dividends paid by the two companies are equally weighted. We identify the optimal value function as the smallest viscosity supersolution of the corresponding Hamilton-Jacobi-Bellman equation and provide an iterative approach to approximate it numerically. Curve strategies are identified as the natural analogue of barrier strategies in this two-dimensional context. A numerical example is presented for which such a curve strategy is indeed optimal among all admissible dividend strategies, and for which this collaboration mechanism outperforms the suitably weighted optimal dividend strategies of the two standalone companies. See [3] for the comparison between merger and stand-alone companies in the diffusion setting, [1] for the performance of another pre-defined risk and profit sharing arrangement, [2] for the study of the dividend problem for two insurance companies that divide between them both claims and premia in some specified proportions under some special dividend strategy and [4] for the study of the survival probability (ruin occurs when neither company can cover the deficit of the other) for collaborating companies.

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