## Evaluating variable annuities with embedded GMWB riders in the presence of withdrawals driven by non-financial factors

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Keywords: Variable Annuities, Guaranteed Minimum Withdrawal Benefit

## Abstract

We consider the problem of evaluating at fair rates a variable annuity (VA) with an embedded guaranteed minimum withdrawal benefit (GMWB) when the withdrawal strategy is influenced by external non-financial factors. When building up a model to evaluate the cost of a GMWB rider, the assumption regarding to the withdrawal strategy of the policyholder plays a crucial role because it has a tremendous impact on the value of the rider. One could be tempted to assume that the policyholder behaviour is optimal and that the withdrawn amount maximizes a certain function, for example the current policy value. This is called the "dynamic strategy". In this case, a problem arises because the assumption that a policyholder is always able to choice the best among a lot of different alternative strategies is not realistic. On the other hand, that policyholders choose withdrawal strategies not always optimal is confirmed also by the market practice, and it is recognized also in actuarial literature.

Among alternative suboptimal withdrawal strategies, the simplest one is to assume that a policyholder never surrenders the contract and that he/she makes constant withdrawals equal to the maximum penalty-free amount. This is often called the "static strategy". In between the static and the dynamic strategy there is the "mixed" strategy under which the policyholder behaves passively with respect to periodical withdrawals but he/she behaves optimally about the decision to surrender the contract. Other possible suboptimal withdrawal strategies have been considered and assume that the withdrawn amount depends upon the moneyness of the guarantee. Both optimal and suboptimal withdrawal strategies considered in literature share the common feature to be driven by financial factors. In reality, many other factors aside from financial convenience influence and may force a policyholder to withdraw or to surrender the policy when it is not optimal. In fact, life insurance policies are long term contracts and adverse events such as unemployment, unexpected expenses and medical bills may cause the need for additional funds that a policyholder could obtain only by taking money from the fund in which the policy premiums have been invested.

We assume that the policyholder may decide to make partial or full withdrawals depending on external non-financial factors. This is done by introducing an hazard rate that determines the probability of such withdrawals whose size is modeled by a suitable chosen random variable. At the same time, the proposed model allows for the possibility of considering optimal or suboptimal withdrawal strategies driven only by economic considerations. Hence, at each date where the policyholder may withdraw, with a certain probability the reference fund may jump down by a random amount due to non-financial factors, while with the remaining probability the jump is induced by an optimal or suboptimal financial withdrawal strategy.

The model is implemented by means of a lattice that discretizes the evolution of the reference fund value along the policy lifetime. Because the decision to withdraw makes the evaluation problem strongly path-dependent, it is not possible to keep track of all the possible reference fund values. To overcome this obstacle a set of "representative" reference fund values is associated with each node of the lattice. Then, starting from maturity, the policy value is computed through the usual backward induction scheme. Once the policy value at inception is obtained, the numerical solution of a non-linear equation furnishes the fair value of the insurance fee. Numerical results highlight the impact of alternative choices for the variables influencing the evaluation model.

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