Pricing and Hedging GLWB in the Heston and Black-Scholes with stochastic interest rates models

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Abstract

Valuing Guaranteed Lifelong Withdrawal Benefit (GLWB) has attracted significant attention from both the academic field and real world financial markets. As remarkerd by Forsyth and Vetzal [2] the Black and Scholes framework seems to be inappropriate for such long maturity products. They propose to use a regime switching model. Alternatively we propose here to use stochastic volatility model (Heston model) and Black-Scholes model with stochastic interest rate (Black-Scholes Hull-White model).

For this purpose we present five numerical methods for pricing GLWB variables annuities: a hybrid tree-finite difference method and a hybrid Monte Carlo method [1], an ADI finite difference scheme, a Fourier cosine method and a Longstaff-Schwartz Monte Carlo method.

These methods are used to determine the no-arbitrage fee for the most popular versions of the GLWB contract, and to calculate the Greeks used in hedging. Both constant withdrawal and optimal withdrawal (including lapsation) strategies are considered.

Numerical results are presented which demonstrate the sensitivity of the no-arbitrage fee to economic, contractual and longevity assumptions.

Keywords — Variables Annuities, GLWB, Finite Difference, Tree methods, Fourier-cosine.

References

[1] M. Briani, L. Caramellino, A. Zanette (2015): Numerical approximations for Heston-Hull-White type models. Preprint, arXiv:1503.03705

[2] P.Forsyth K.Vetzal (2014): An optimal stochastic control framework for determining the cost of hedging of variable annuities. *Journal of Economic Dynamics and Control* 44 (2014) 29-53.

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