Expansions of renewal functions and applications to ruin probabilities

Clément Dombry, Landy Rabehasaina

Laboratory of Mathematics, Franche Comté University, 16 route de Gray, 25030 Besançon, France.
E-mail addresses: clement.dombry@univ-fcomte.fr; lrabehas@univ-fcomte.fr
Phone: +33 3 81 66 63 00

Abstract Let $(X_k)_{k \in \mathbf{N}}$ be an i.i.d. sequence of non negative random variables with common cumulative distribution F. We consider the associated renewal function $U(x) := \sum_{n=1}^{\infty} F^{*(n)}(x)$, $x \geq 0$. It is known that if X_1 has a second moment and is non lattice, then one has the following expansion

$$U(x) = \frac{x}{\mu} + \frac{\mu_2}{2\mu^2} + o(1) \quad \text{as } x \to \infty, \tag{1}$$

with $\mu = E(X_1)$ and $\mu_2 = E(X_1^2)$ the first and second moment respectively. Our aim of this talk is twofold. First, we provide additional terms in the $o(1)$ term in (1) when the X_k's are light tailed, using mainly an approach by [4], based on complex analysis. Then, we use this tool to obtain high order expansions of ruin probabilities as initial reserves tend to infnity and when claims are light tailed, which enables us to recover and extend some results from [2]. Discrete versions of these results are also discussed. As a by-product of this, we consider a two dimensional risk process, and we obtain a two-terms asymptotic expansion as the initial reserves tend to infinity along a fixed direction, which generalizes some results from [1].

Keywords: renewal functions, risk theory, expansions.

References

[1] Avram, F., Palmowski, Z. and Pistorius, M. (2008). Exit problem of a two-dimensional risk process from the quadrant: Exact and asymptotic results. *Annals of Applied Probability* 18(6): pp.2421–2449.

[2] Kuznetsov A. and Morales M. (2014). Computing the finite-time expected discounted penalty function for a family of Lévy risk processes. *Scandinavian Actuarial Journal*, 2014(1), pp.1–31.

[3] Li S., Lu Y. and Garrido J. (2009). A review of discrete-time risk models. *Journal Serie A Matemáticas (RACSAM)*, 103(2), pp.321–337.

[4] Stone C. (1965). On characteristic functions and renewal theory. *Transactions of the American Mathematical Society*, 120(2), pp.327–342.