# Optimal Dividends in the Dual Model under Fixed Transaction Costs

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## Abstract

We solve the optimal dividend problem under fixed transaction costs in the so-called dual model, in which the surplus of a company is driven by a Lévy process with positive jumps (spectrally positive Lévy process). This is an appropriate model for a company driven by inventions or discoveries. The case without transaction costs has recently been well-studied. In particular, in Bayraktar et al. (2013), we show the optimality of a barrier strategy (reflected Lévy process) for a general spectrally positive Lévy process of bounded or unbounded variation.

We show that a $(c_1^*, c_2^*)$-policy is optimal for some $c_2^* > c_1^*$. For $c_2 > c_1$, a $(c_1, c_2)$-policy brings the level of the controlled surplus process down to $c_1$ whenever it reaches or exceeds $c_2$.

In order to derive this result, we first write the corresponding net present value using the scale function. We then show the existence of the maximizers $c_1^* < c_2^*$ that satisfy the continuous fit (resp. smooth fit) at $c_2^*$ when the surplus process is of bounded (resp. unbounded) variation and that the derivative at $c_1^*$ is one when $c_1^* > 0$ and is less than or equal to one when $c_1^* = 0$. These properties are used to verify the optimality of the $(c_1^*, c_2^*)$-policy. The levels $c_1^*$ and $c_2^*$ as well as the value function are written succinctly in terms of the scale function.

**Keywords:** dual model; dividends; impulse control; spectrally positive Levy processes; scale functions