

## **Valuation of Guaranteed Minimum Maturity Benefits in variable annuities with surrender options**

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### **Abstract**

We consider the pricing of guaranteed minimum maturity benefits (GMMB) embedded in variable annuity contracts in the case where the guarantees can be surrendered any time prior to maturity. Surrender charges are imposed as a way of discouraging early termination of the variable annuity contract. We formulate the problem as an American put option and derive the corresponding pricing partial differential equation (PDE) using hedging arguments and Ito's Lemma. Given the underlying stochastic evolution of the fund, we also present the associated transition density PDE whose solution is well known in literature. An explicit integral expression for the pricing PDE is then presented with the aide of Duhamel's principle. An expression for the delta of the surrender option which can be used for risk management purposes is also derived. We then outline the algorithm for implementing the integral expression for the price and the corresponding early exercise boundary for the surrender option. We wrap up the paper by presenting numerical results for the prices, early exercise boundaries, deltas and the corresponding sensitivities.

Keywords: GMAB, Surrender options, numerical integration