Infinite-time Absolute Ruin in Dependent Renewal Risk Model with Constant Force of Interest

Jiajun Liu[a](Speaker) Yang Yang[b,c] *

Abstract

Consider a renewal risk model with a constant premium and a constant force of interest rate, where the claim sizes and inter-arrival times follow certain dependence structures via some restriction on their copula function. Under the assumption that the distribution of the claim-size belongs to the intersection of the class $\mathcal{S}(\gamma)$, $\gamma \geq 0$ and the class $\mathcal{R}_{-\infty}$, or a larger intersection class of O-subexponential distribution, class $\mathcal{L}(\gamma)$ and $\mathcal{R}_{-\infty}$, the infinite-time absolute ruin probabilities are derived.

Keywords: Asymptotics; Class $\mathcal{S}(\gamma)$; Class $\mathcal{L}(\gamma)$; Dependence; Constant force of interest; Farlie-Gumbel-Morgenstern distribution; Heavy tails; O-subexponential distribution class; Renewal risk model; Ruin probability.

1 Introduction

Consider the following renewal risk model with constant premium rate and constant force of interest. Denote by $W_r(t)$, the total reserve up to time t, satisfies

$$W_r(t) = x\mathrm{e}^{rt} + c\int_0^t \mathrm{e}^{r(t-y)}\mathrm{d}y - \sum_{i=1}^{N(t)} X_i \mathrm{e}^{r(t-\tau_i)}, \qquad t \geq 0.$$

Inspired by the literature, we define the probability of infinite-time absolute ruin as

$$\psi(x,\infty) = \mathrm{Pr}\left(\inf_{t\geq 0} W_r(t) < -\frac{c}{r} \middle| W_r(0) = x \right), \qquad x \geq 0, \tag{1.1}$$

which can also be rewritten as

$$\psi(x,\infty) = \mathrm{Pr}\left(\sum_{i=1}^{\infty} X_i \prod_{j=1}^{i} Y_j > x + \frac{c}{r} \right), \qquad x \geq 0, \tag{1.2}$$

where $Y_i = \mathrm{e}^{-r\theta_i}, i \geq 1$, can be considered as discount factor according to the constant force of interest r.

Definition 1.1 *The corresponding survival copula is defined as*

$$\overline{C}(u,v) = u + v - 1 + C(1-u, 1-v), \qquad (u,v) \in [0,1]^2.$$

Assume that the copula function $C(u,v)$ is absolutely continuous, denote by $C_1(u,v) := \frac{\partial}{\partial u}C(u,v)$, $C_2(u,v) := \frac{\partial}{\partial v}C(u,v)$, and $C_{12}(u,v) := \frac{\partial^2}{\partial u \partial v}C(u,v)$, then

$$\overline{C}_2(u,v) := \frac{\partial}{\partial v}\overline{C}(u,v) = 1 - C_2(1-u,1-v), \overline{C}_{12}(u,v) := \frac{\partial^2}{\partial u \partial v}\overline{C}(u,v) = C_{12}(1-u,1-v).$$

Assumption 1.1 *The relation*

$$\overline{C}_2(u,v) \sim u\overline{C}_{12}(0+,v), \qquad u \downarrow 0,$$

holds uniformly on $(0,1]$.

*[a]Department of Mathematical Sciences, University of Liverpool, Peach Street, Liverpool, L69 7ZL, United Kingdom. E-mail: J.Liu9@liverpool.ac.uk

2 Main results

Theorem 2.1 *In the renewal risk model with constant force of interest rate* $r > 0$*, assume that* (X_i, θ_i)*,* $i \in \mathbb{N}$*, are a sequence of i.i.d. random pairs with generic random pair* (X, θ) *satisfying Assumption* **??** *with* $F \in \mathcal{S}(\gamma) \cap \mathcal{R}_{-\infty}$ *for some* $\gamma \geq 0$*, then* $\mathrm{E}\mathrm{e}^{\gamma S_\infty} < \infty$*, where* $S_\infty = \sum_{i=1}^{\infty} X_i \mathrm{e}^{-r\tau_i}$*, and*

$$\psi(x, \infty) \sim \mathrm{e}^{-\gamma c/r} \mathrm{E}\mathrm{e}^{\gamma S_\infty} \int_0^\infty \overline{F}(x\mathrm{e}^{rt}) G_{\theta_c}(\mathrm{d}t), \tag{2.1}$$

where $G_{\theta_c}(\mathrm{d}t) = C_{12}\,[1-, G_\theta(t)]\, G_\theta(\mathrm{d}t)$*.*

Theorem 2.2 *In the renewal risk model with constant force of interest rate* $r > 0$*, assume that* (X_i, Y_i)*,* $i \in \mathbb{N}$*, are a sequence of i.i.d. random pairs with generic random pair* (X, Y) *under Assumption* **??** *with* $F \in \mathcal{S}(\gamma) \cap \mathcal{R}_{-\infty}$ *for some* $\gamma \geq 0$*, then* $\mathrm{E}\mathrm{e}^{\gamma S_\infty} < \infty$*, where* $S_\infty = \sum_{i=1}^{\infty} X_i \prod_{j=1}^{i} Y_j, \quad Y_j = \mathrm{e}^{-r\theta_i}$*, and*

$$\psi(x, \infty) \sim \mathrm{E}\mathrm{e}^{\gamma S_\infty} \Pr(XY_c > x + c/r), \tag{2.2}$$

where Y_c *is distributed by* $G_c(\mathrm{d}y) = C_1[1-, G(\mathrm{d}y)] = C_{12}[1-, G(y)]G(\mathrm{d}y)$*.*

Theorem 2.3 *In the renewal risk model with constant force of interest rate* $r > 0$*, assume that* (X, θ) *or* (X, Y) *is dependent according to Assumption* **??** *with* $F \in \mathcal{L}(\gamma) \cap \mathcal{OS} \cap \mathcal{R}_{-\infty}$ *for some* $\gamma \geq 0$*, then relation (**??**) or (**??**) holds with* $\mathrm{E}\mathrm{e}^{\gamma S_\infty} < \infty$*.*

Theorem 2.4 *In the renewal risk model with constant force of interest rate* $r > 0$*, assume that* (X_i, Y_i)*,* $i \in \mathbb{N}$*, are a sequence of i.i.d. random pairs with generic random pair* (X, Y) *following a common bivariate FGM distribution function with* $\rho \in [-1, 1]$*. If* $F \in \mathcal{S}(\gamma) \cap \mathcal{R}_{-\infty}$ *for some* $\gamma \geq 0$*, then* $\mathrm{E}\mathrm{e}^{\gamma S_\infty} < \infty$*, where* $S_\infty = \sum_{i=1}^{\infty} X_i \prod_{j=1}^{i} Y_j$*, and*

$$\psi(x, \infty) \sim \mathrm{E}\mathrm{e}^{\gamma S_\infty} \Pr(XY_\rho > x + c/r),$$

where Y_ρ *is distributed by* G_ρ *with* $G_\rho(y) = (1-\rho)G(y) + \rho G^2(y)$*.*

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