Pricing Asian options in a general tree model with varying skewness and kurtosis

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We propose a model for pricing European-type Asian options based on the arithmetic average price of the underlying asset. Arithmetic averaging causes nontrivial complexity in the pricing problem and no exact closed-form formula exists; a problem that has challenged both financial academia and practitioners for many years. Our approach relies on a bivariate tree when the underlying process is a jump-diffusion (see Hilliard and Schwartz, 2005) which ensures that skewness and kurtosis do not remain fixed; a bivariate tree is constructed for the smooth diffusion and for the jumps under Poisson compounding. The proposed method is easily adaptable to the case of a multinomial lattice model (see Černý, 2009) and the case of a forward-start option based on a deferred average price (e.g., see Reynaerts et al., 2006) as well as the option price sensitivities.

We demonstrate the high efficiency of our method by means of comparisons with Monte Carlo price estimates and other published results, including Hull and White (1993), Chalasani et al. (1998), Neave and Ye (2003), Hsu and Lyuu (2007) and Lo et al. (2008). We test our method on both discretely and continuously sampled Asian options of European type. Future research is targeted towards extending to pricing Asian options of American type.

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