Combined Estimation-Optimization (CEO) Approach for High Dimensional Portfolio Selection

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ABSTRACT

We propose a combined estimation-optimization (CEO) approach to highdimensional portfolio selection problems in which the number of risky assets (*p*) is greater than the number of observation times (n). The spirit of the CEO approach lies on constructing a direct estimation to the theoretical optimal trading strategy (optimal control), instead of separating the estimation and optimization procedures. We investigate a constrained l_1 -minimization for estimating the optimal control and apply it to the mean-variance portfolio (MVP) problems under single-period (static) and multi-period (dynamic) settings. We prove that plugging the sample variance-covariance matrix into the optimal MVP strategy makes the probability that the optimal trading strategy outperforms the bank account tend to 50% for p >> n and a large *n*, even though the mean vector is estimated without estimation error. The CEO approach converges to the true optimal solution for a large n for all of the aforementioned model settings. The distinctive advantages of the CEO approach over its competitive methods are the simple implementation scheme, the guarantee of existence of solution even when the sample variance-covariance matrix is singular, the application beyond Gaussian distribution of stock returns, and the application beyond static model. In addition, the CEO scheme automatically filters out unfavorable stocks based on historical data. Simulations validate the theory and the behavior of the proposed approach. Empirical studies show that the CEO-based portfolio outperforms the equal-weight portfolio, the MVP with shrinkage estimators and other competitive approaches.

Keywords: High-dimensional portfolio; l_1 -minimization; Sparse trading strategy.

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