## Application of Metamodeling in Variable Annuity Portfolio Valuation

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## Abstract

Variable annuities are life insurance policies that contain complex guarantees. Due to the complexity of the guarantees, there is no closed-form formula to calculate the value of the guarantees except for some special cases. Insurance companies rely heavily on the Monte Carlo simulation method to price the guarantees. However, it is very time-consuming to use the Monte Carlo simulation method to price a large portfolio that contains hundreds of thousands of variable annuity policies. For example, if we use a Monte Carlo simulation model with 1,000 risk-neutral scenarios and 360 monthly time steps to value a portfolio of 100,000 variable annuity policies, then the number of cash flow projections is  $3.6 \times 10^{10}$ . It will take a computer that can process 200,000 cash flow projections per second about 50 hours to finish the computation. As a result, valuation and risk management of such large portfolios are a big challenge to insurance companies.

We propose to use a metamodeling approach to address the computational challenge mentioned above. The metamodeling approach consists of two components: an experimental design method and a metamodel. The experimental design method is used to select a small set of variable annuity policies from a large portfolio. For example, Latin hypercube sampling is a popular experimental design method. The metamodel is used to approximate the fair market value (or other quantities of interesting) of the portfolio based on the selected policies and their fair market values calculated by the Monte Carlo simulation model. Since the Monte Carlo simulation model is used to value only a small number of selected policies and the metamodel is fast, the metamodel approach is able to speed up the valuation of large portfolios significantly and produce accurate approximations. In practice, kriging is a metamodel that works well for variable annuity. We applied a metamodeling approach with Latin hypercube sampling and kriging to value a portfolio of 200,000 synthetic variable annuities, the approach is able to approximate the fair market value of the portfolio with a percentage error of 0.42% and reduce the running time from 1942.22 seconds to 12.51 seconds.

Keywords: Variable annuity, Portfolio valuation, Metamodeling, Latin hypercube sampling, Kriging

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