Prediction Intervals with ARIMA Processes

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Abstract

One purpose of risk modelling is to produce prediction intervals, in which future observations will fall, with a certain probability, given what has already been observed. Prediction intervals form the theoretical underpin value-at-risk calculations.

Time series theory already provides forecasting equations, describing the distribution of future observations given past history and estimated parameters. These methods have become embedded in actuarial practice, particularly for asset-liability modelling, and are especially useful when historic datasets are large enough that parameter estimation error is not material.

Where data is limited in quantity or quality, we encounter a different set of problems. Given a historic data set oscillating within a narrow range, calibration tools may fit a model reverting rapidly to the middle of the historic range. However, them same data could nearly as plausibly have come from a process which reverts slowly to a distant value well outside the historic range. Prediction intervals allowing for randomness in parameter estimates, as well as in future random innovations, can be far wider than implied by forecasting equations that ignore parameter error. There is a legitimate concern that existing forecasting methods for time series models may understate the risk of some investment strategies and skew the results of asset-liability models.

Our approach to prediction interval construction reduces to the standard prediction intervals based on Student's T distribution in the special case of regression problems. In the more general case, the

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non-linear nature of autoregressive recursion precludes analytical treatment. Instead, we use Monte Carlo tools to construct prediction intervals for integrated auto-regressive moving average (ARIMA) processes, whose parameters are unknown.

We illustrate the impact of the proposed techniques by comparing forecast intervals to prediction intervals for a range of ARIMA models, including some based on Wilkie's stochastic economic model.

Keywords: Time Series, Market Risk, Inflation, Forecasting, Prediction Interval, Back-Testing, Monte Carlo Simulation.