On Hoeffding's Covariance Lemma: Its Functional Generalizations and Applications

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Abstract

We review, extend and apply Hoeffding's covariance lemma, which is a lesser-known technical result that lends itself to the study of dependence structures and stochastic orders. In the first part, we generalize Hoeffding's lemma to the covariance between appropriate transformations of random variables. While similar functional generalizations can be found in the literature, most overlook the possible discontinuities of the transformations and the subtle definition of Lebesgue-Stieltjes integral, and thus fail to hold in general. Armed with the correctly formulated functional generalizations, we present in the second part a host of applications of the generalized Hoeffding's lemma. These applications highlight the theoretical and practical utility of Hoeffding's lemma, as well as its connections to diverse research areas.

Keywords Lebesgue-Stieltjes integral; Correlation order; Quadrant dependence; Comonotonicity; Marshall-Olkin exponential distribution