

On the Optimal Reinsurance Problem

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Abstract

Since Borch and Arrow published their celebrated seminal papers, the optimal reinsurance problem has been addressed by many authors and under many different risk measurement methods and premium principles.

Usually, authors assume that the statistical distribution of claims is known. Nevertheless, measurement errors or lack of complete information may provoke discrepancies between the real and the estimated probabilities of the states of nature, generating uncertain (also called ambiguous) frameworks.

This study addresses the optimal reinsurance problem with two novelties. The most important one is the incorporation of uncertainty, i.e., insurer and reinsurer are not sure about the real probabilities of every state of nature. Furthermore, insurer and reinsurer may reflect different uncertainty levels.

The second novelty is related to the moral hazard of the reinsurer. We will address the topic under two different perspectives. In a first approach the insurer decision variable will be the mathematical derivative of the retained risk with respect to the global claims, rather than the retained risk itself. With this modification the reinsurer can impose positive lower bounds to this decision variable, and therefore contracts reflecting spreads with null derivative (flat behavior of the retained risk with respect to the global claims) become unfeasible. In other words, the usual reinsurer moral hazard is totally eliminated. Nevertheless, if insurer and reinsurer trust each other, or they have other methods (accounting inspections, for instance) so as to control moral hazards, then we

can select the retained risk itself as a decision variable (instead of its derivative). With this second and alternative approach there is also a critical difference between our analysis and previous literature, since we will not impose comonotonic costs for insurer and reinsurer. In fact, we will show how comonotonicity may become a significant limitation in practice, and the lack of comonotonicity constraints may allow the insurer to reduce the global risk in a much more efficient manner. Therefore, comonotonicity may be inefficient if there are alternative procedures to deal with the moral hazard. Summarizing, our second novelty is the total elimination of the moral hazard (first approach) or the total elimination of comonotonicity if this provokes an efficiency reduction of the optimal contract and there are alternative methods for the moral hazard control.