A study of dividends and optimal barriers for a dual risk model

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Abstract

The dual risk model assumes that the surplus of a company decreases at a constant rate over time, and grows by means of upward jumps which occur at random times and at random sizes. For companies with economical activities involved in research and development, these jumps can be interpreted as the values of future gains from an invention or discovery, and the decrease of surplus, at a constant rate, can represent costs of production, payments to employees, maintenance of equipment, etc.

The name *dual* is derived from the original, or *primal*, model: the Cramèr–Lundberg insurance risk model. In the latter the surplus increases at a constant rate and the jumps are downwards.

Recent works on the dual model include those by , Avanzi et al (2007), Cheung and Drekic (2008), Ng (2009), Afonso *et al*. (2013), Rodríguez–Martínez *et al*. (2015), among others.

Most of the above authors have worked the dual model in the case of exponentially distributed waiting times. Rodríguez–Martínez *et al*. (2015) generalized to the case of Erlang(n) waiting times.

We generalize further to compound renewal dual risk models where the waiting times are Phase–Type distributed. Using the roots of the fundamental and the generalized Lundberg's equation, we perform calculations to obtain the expected discounted dividends and higher moments, assuming that the gains follow as well a Phase–Type distribution.

We also develop a numerical method to determine the dividend barrier that maximizes the expected discounted dividends.

Finally, we perform illustrations working some examples for some particular gain distributions and obtain numerical results.

Keywords: Dual risk model; Phase–Type distribution, generalized Lundberg's equation; discounted dividends.

References

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