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The 2-variable polynomial of cable knots

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Abstract

The 2-variable polynomial P_K of a satellite K is shown not to satisfy any formula, relating it to the polynomial of its companion and of the pattern, which is at all similar to the formulae for Alexander polynomials. Examples are given of various pairs of knots which can be distinguished by calculating P for 2-strand cables about them even though the knots themselves share the same P. Properties of a given knot such as braid index and amphicheirality, which may not be apparent from the knot's polynomial P, are shown in certain cases to be detectable from the polynomial of a 2-cable about the knot.

1. *Introduction*

With the development of the 2-variable polynomial P_K of an oriented knot K (of one or more components) [**5**], has come the desire to relate it directly to the geometry of the knot exterior, as Seifert, and later Fox, were able to do with the Alexander polynomial. One obvious place where the geometry should show up is when the knot is a satellite. In this case the exterior is made up of the union of two link exteriors, that of a *companion* and that of a *pattern link*, forming part of a natural decomposition of the exterior in the context of 3-manifolds.

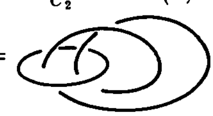
The multi-variable Alexander polynomial of such a union bears a simple relation to those of the two constituent link exteriors. This takes the form of an equation $S_K = S_C S_R$, where K is a satellite with companion C and pattern link R consisting of an unknotted component defining a complementary solid torus V which contains the other component(s) of R. The knot K is formed from the image of these component(s) when a solid torus neighbourhood of C is replaced by V using a faithful (longitude-preserving) homeomorphism. In the equation S is either the multi-variable Alexander polynomial, or, in the case of a 1-component knot, a modified version of it; the variables, which correspond to homology classes in the appropriate link exterior, are each replaced by the element which they represent in the exterior of K. See [**2**] for a general, but not very readable, account.

Possible satellite formula

On the analogy of the Alexander polynomial, we expected some such satellite formula for the other polynomials. For each K a conjectural function S_K of several variables might exist, specializing to P_K in some way and satisfying an equation $S_K = S_C S_R \phi$ for a satellite K of C with pattern R, where the variables in S_C and S_R are substituted by others in a way which depends in some suitable sense only on the gluing of these two exteriors, and ϕ is a possible normalizing factor, again depending only on the gluing homeomorphism.

Under such a framework, if two knots C_1 and C_2 with $S_{C_1} = S_{C_2}$ were used with the same pattern and method of gluing to construct satellites K_1 and K_2 we would then have $S_{K_1} = S_{K_2}$.

Conceivably there may be an S_K which specialises to P_K and satisfies such a satallite formula (possibly specializing also to Kauffman's recently announced polynomial F_K [**8**]). In this paper we show however that no such function S_K can be found with the additional property that $S_K = P_K$ for a 1-component knot K, even if it differs from P_K when K has more components.

The examples given are pairs of knots C_1 and C_2 with $P_{C_1} = P_{C_2}$ whose (2, 1) cables K_1 and K_2 (satellites constructed with the same pattern $R =$ ) have $P_{K_1} \neq P_{K_2}$. These are described in a later section under the heading 'Birman's pairs of 3-braids' and the polynomials are displayed in Table 1.

2. *Scope of the calculations*

To compute the polynomial P for these examples we developed a program to calculate P_K from a presentation of K as a closed braid, based on the construction of Ocneanu and Jones [**13**, **6**]. Details of the theoretical basis for the calculations, and their practical implementation are given in [**12**].

Our program can handle interactive calculations for 7-string braids of over 100 crossings and will deal with 8-string braids of up to 150 crossings on the Liverpool University IBM 3083 computer using less than 16 megabytes storage and under 200 seconds of computer time. The time required grows relatively slowly (quadratically) with the number of crossings for braids of a given string index, so within the constraints of presentation as a closed braid on at most 8 strings the method provides an efficient way of handling knots with many crossings. It may be contrasted with Thistlethwaite's encyclopaedic work in producing tables of P for all knots up to 13 crossings based on the Conway recurrence relation [**14**]. His method works well, given information about all knots with fewer crossings, and is not particularly sensitive to braid index, but in general it would face exponential time growth with the number of crossings if calculation of P for an individual knot beyond the range of the table was required.

Annotated copies of the Pascal program to calculate the polynomial P and also the Alexander and Jones polynomials of a knot presented as a closed braid on at most eight strings are available on request.

3. *Further consequences*

While the failure of the satellite formula proves disappointing from the point of view of understanding the general structure of the polynomial P_K, it leaves the way open to using P in a second attempt at distinguishing two knots C_1 and C_2 with $P_{C_1} = P_{C_2}$ by comparing P on satellites of C_1 and C_2. From our limits on the computing power available, we have been restricted to considering 2-string cables where C_1 and

C_2 can themselves be presented as closed braids on at most four strings, so that the cables can be presented as 8-string braids. Other satellites such as doubles or higher string cables could in principle be used.

The only examples so far observed where the satellites are still indistinguishable using P have come when C_2 is a mutant of C_1. In all cases tried so far, if C_1 and C_2 are mutants then their 2-cables, although not apparently mutants themselves, have the same P. Examples are discussed in a later section.

Burau polynomials

The possibility of calculating P_K from the 'Burau polynomial' of a closed braid representative β of K, i.e. from the characteristic polynomial $\det(xI-\beta(t))$ of the Burau matrix $\beta(t)$ for β, was raised tentatively by Jones[7]. Our examples used in disproving the existence of a simple satellite formula can also serve to discount this possibility.

To see why this is so, suppose that C is a knot presented as the closure $\hat{\beta}$ of some $\beta \in B_n$. The *complete closure*, $\hat{\beta} \cup L_\beta$, consisting of $\hat{\beta}$ together with the braid axis L_β is then a link whose 2-variable Alexander polynomial is the Burau polynomial of β [11]. The $(2, r)$ cable K about C can be presented naturally as the closure of a $2n$-string braid, by doubling all strings in β, with twists as required. The complete closure of this braid is then a satellite constructed using the $(2, r)$ cable pattern R on the string C from the complete closure $C \cup L_\beta$ of β. Using the satellite formula for Alexander polynomials the Burau polynomial of this representation for K can then be calculated from that of $C \cup L_\beta$ and the pattern.

If this construction is applied to two braids β_1 and β_2 with the same Burau polynomial, the resulting braid presentations for the $(2, r)$ cables K_1 and K_2 will then have the same Burau polynomial. The examples of Birman given in tables 1 and 2 are $(2, r)$ cables about 3-braids with the same Burau polynomial. The resulting 6-braids have then the same Burau polynomials, but their closures K_1 and K_2 have $P_{K_1} \neq P_{K_2}$.

With the failure of the satellite formula, features of a knot K which do not show up directly from P_K may nevertheless become apparent from the polynomial of a 2-cable about K. These features include the *braid index* and the question of *amphicheirality* of K, and examples are discussed in the course of the next section.

4. *Discussion of examples*

Braids in the accompanying tables are listed as elements of B_8, using $\pm i$ to stand for the generator $\sigma_i^{\pm 1}$. The polynomial $P_K(v, z)$ is given as a matrix of coefficients (p_{ij}), where $P_K(v, z) = \Sigma p_{ij} z^i v^j$, with the range of i and j indicated at the side. We use the convention that $v^{-1}P_{K^+} - vP_{K^-} = zP_{K^0}$, where K^+, K^- and K^0 differ only as shown:

K^+ K^- K^0

Putting $v = 1$ gives $\nabla_K(z)$, the Conway polynomial. The substitutions ($z = x - x^{-1}$, $v = 1$) and ($z = x - x^{-1}$, $v = x^2$) give respectively the Alexander and Jones polynomials, $\Delta(x^2)$ and $V(x^2)$. In the tables the negative powers of x in the Alexander polynomial have been omitted, because of its invariance when x is replaced by $-x^{-1}$.

Table 1

Braid representing the 2-cable K_1

*** -2-1-3-2(4354)^{7}167 ***

The polynomial P for K_1: 24	26	28	30		Conway polynomial	Alexander polynomial	Jones polynomial
50	−90	45	−4	0	1 (0)	1 (0)	1 (24)
750	−1140	480	−45	2	45 (2)	−1 (6)	1 (28)
4445	−5383	1709	−111	4	660 (4)	1 (8)	1 (32)
14105	−13320	3002	−113	6	3674 (6)	−1 (14)	−1 (34)
27092	−19723	3003	−54	8	10318 (8)	1 (16)	1 (36)
33566	−18654	1820	−12	10	16720 (10)	−1 (22)	−1 (38)
27798	−11643	680	−1	12	16834 (12)	1 (24)	−1 (54)
15656	−4846	153		14	10963 (14)		−1 (58)
6004	−1330	19		16	4693 (16)		1 (60)
1541	−231	1		18	1311 (18)		−1 (62)
253	−23			20	230 (20)		1 (64)
24	−1			22	23 (22)		−1 (66)
1				24	1 (24)		1 (68)

Braid representing the 2-cable K_2

*** $(4354)^2(2132)^2(4354)^2(2132)^2 4354(2132)^{-3}167$ ***

The polynomial P for K_2

20	22	24	26	28	30		Conway polynomial	Alexander polynomial	Jones polynomial
−16	80	−110	70	−35	12	0	1 (0)	1 (0)	−1 (18)
−300	1220	−1210	420	−140	55	2	45 (2)	−1 (6)	1 (20)
−1874	7076	−5467	1029	−217	113	4	660 (4)	1 (8)	1 (22)
−5806	21558	−13365	1340	−166	113	6	3674 (6)	−1 (14)	1 (28)
−10363	39414	−19734	1013	−66	54	8	10318 (8)	1 (16)	1 (30)
−11452	46372	−18655	456	−13	12	10	16720 (10)	−1 (22)	−1 (34)
−8127	36484	−11643	120	−1	1	12	16834 (12)	1 (24)	1 (36)
−3757	19549	−4846	17			14	10963 (14)		−1 (38)
−1123	7145	−1330	1			16	4693 (16)		−1 (42)
−209	1751	−231				18	1311 (18)		1 (44)
−22	275	−23				20	230 (20)		−1 (46)
−1	25	−1				22	23 (22)		1 (48)
	1					24	1 (24)		−1 (54)
									−1 (62)
									1 (64)
									−1 (70)
									1 (72)

Table 2

Braid representing a knot C_1
*** 11122221211-2-2-2111234567 ***
Braid representing a knot C_2
*** 111222212111211-2-2-234567 ***

The polynomial P for C_1 and C_2

10	12	14	
5	−3	−1	0
31	−11	−4	2
52	−9	−4	4
35	−2	−1	6
10			8
1			10

Braid representing the (2, 1)-cable about C_1
*** $(2132)^3(4354)^4 21324354(2132)^2(4354)^{-3}(2132)^3 43541^{-23}67$ ***

The polynomial P for this cable

20	22	24	26	28	30	
−149	407	−319	46	−3	19	0
−4382	12124	−9880	2415	−1076	863	2
−50472	140079	−114785	34243	−21388	12963	4
−317466	886465	−716356	234730	−183608	98595	6
−1251252	3544185	−2787908	963172	−918054	453992	8
−3336727	9681064	−7332716	2624764	−3019960	1387543	10
−6321130	18982322	−13699429	5041481	−6971998	2971026	12
−8788244	27628767	−18789643	7085256	−11748376	4613040	14
−9165353	30555075	−19345884	7466144	−14823467	5313655	16
−7272714	26086910	−15171599	5991977	−14243411	4608857	18
−4426196	17366982	−9138087	3695028	−10534332	3036606	20
−2071246	9059325	−4238499	1755890	−6030207	1524737	22
−742861	3702454	−1509149	641028	−2673527	582055	24
−202189	1179084	−408411	178007	−913881	167390	26
−40980	289138	−82424	36889	−238235	35612	28
−5986	53481	−12004	5520	−46439	5428	30
−595	7210	−1191	563	−6547	560	32
−36	668	−72	35	−630	35	34
−1	38	−2	1	−37	1	36
	1			−1		38

Conway polynomial	Alexander polynomial	Jones polynomial	
1 (0)	11 (0)	−1 (4)	12 (58)
64 (2)	−10 (4)	1 (6)	−5 (60)
640 (4)	7 (8)	−1 (8)	−5 (62)
2360 (6)	−3 (12)	1 (10)	3 (64)
4135 (8)	1 (20)	−1 (12)	5 (66)
3968 (10)		1 (14)	−4 (68)
2272 (12)		−1 (16)	−10 (70)
800 (14)		1 (18)	14 (72)
170 (16)		1 (22)	−1 (74)
20 (18)		2 (30)	−11 (76)
1 (20)		−2 (34)	7 (78)
		−1 (36)	4 (80)
		5 (38)	−8 (82)
		−9 (42)	4 (84)
		5 (44)	1 (86)
		8 (46)	−3 (88)
		−11 (48)	2 (90)
		−1 (50)	1 (92)
		13 (52)	−2 (94)
		−9 (54)	1 (96)
		−5 (56)	

Braid representing the (2, 1)-cable about C_2

*** $(2132)^3(4354)^4 21324354(2132)^3 4354(2132)^2(4354)^{-3}1^{-23}67$ ***

The polynomial P for this cable							Conway polynomial	Alexander polynomial	Jones polynomial	
20	22	24	26	28	30					
−165	487	−479	206	−83	35	0	1 (0)	11 (0)	−1 (4)	12 (58)
−4682	14144	−15040	8775	−4896	1763	2	64 (2)	−10 (4)	1 (6)	−5 (60)
−52346	156355	−164697	104655	−68114	24787	4	640 (4)	7 (8)	−1 (8)	−6 (62)
−323272	953193	−962106	626410	−464256	172391	6	2360 (6)	−3 (12)	1 (10)	4 (64)
−1261615	3708909	−3525854	2288638	−1930063	724120	8	4135 (8)	1 (20)	−1 (12)	4 (66)
−3348179	9947173	−8810295	5620926	−5434061	2028404	10	3968 (10)		1 (14)	−3 (68)
−6329257	19277235	−15776844	9817955	−11000795	4013978	12	2272 (12)		−1 (16)	−10 (70)
−8792001	27859220	−20907214	12641994	−16620167	5818968	14	800 (14)		1 (18)	14 (72)
−9166476	30683856	−20941807	12287546	−19189381	6326432	16	170 (16)		1 (22)	−1 (74)
−7272923	26138520	−16070255	9153211	−17183061	5234528	18	20 (18)		2 (30)	−11 (76)
−4426218	17381680	−9517127	5270758	−12031334	3322242	20	1 (20)		−2 (34)	8 (78)
−2071247	9062226	−4357579	2352590	−6606731	1620741	22			−1 (36)	3 (80)
−742861	3702831	−1536581	811317	−2840188	605482	24			4 (38)	−8 (82)
−202189	1179113	−412907	213996	−949436	171423	26			1 (40)	4 (84)
−40980	289139	−82920	42346	−243661	36076	28			−8 (42)	2 (86)
−5986	53481	−12037	6081	−46999	5460	30			4 (44)	−4 (88)
−595	7210	−1192	598	−6582	561	32			8 (46)	1 (90)
−36	668	−72	36	−631	35	34			−11 (48)	2 (92)
−1	38	−2	1	−37	1	36			12 (52)	−2 (94)
	1			−1		38			−9 (54)	1 (96)
									−5 (56)	

The Alexander polynomial in particular serves as a useful check for the calculations, since it can be quickly found for a cable using the satellite formula.

It is noticeable in these examples that the coefficients of the Alexander and Jones polynomials are considerably smaller than those in the Conway polynomial. This cannot be true in general, since all integer polynomials in z^2 with constant term 1 are possible as Conway polynomials of some knot, and so some Conway polynomials must correspond to Alexander polynomials with larger coefficients. Most probably it is a result of using fairly positive braids on a relatively small number of strings.

To reduce the size of the coefficients in these examples it would be tempting to rewrite P itself in terms of x rather than z, but this would only be possible for a 1-component knot because of the negative powers of z which occur otherwise.

Birman's pairs of 3-braids

The original motivation for these calculations arose from the simple examples of pairs of 3-braids discovered by M. T. Lozano and H. R. Morton, and simultaneously in greater variety by J. Birman [1]. Each example consists of two 3-braids β_1, β_2, with the same exponent sum and the same trace for their Burau matrix, consequently the same Burau polynomial, while closing to inequivalent knots. Since the polynomial P for the closure of a 3-braid is also determined by exponent sum and trace of Burau matrix these pairs give examples of inequivalent knots with the same polynomial P.

One of the simplest such pairs is $\beta_1 = \sigma_1^{-1}\sigma_2^7$, $\beta_2 = \Delta_3^4\sigma_1\sigma_2^{-7}$, conjugate to $\sigma_2^2\sigma_1^2\sigma_2^2\sigma_1^2\sigma_2\sigma_1^{-3}$, where Δ_n is the half twist on n strings, so that $\Delta_3 = \sigma_1\sigma_2\sigma_1$. Their closures give inequivalent knots C_1 (the (2,7) torus knot) and C_2. A 2-cable about each can be presented as a 6-braid, by replacing σ_1 with $\sigma_2\sigma_3\sigma_1\sigma_2$ and σ_2 with $\sigma_4\sigma_5\sigma_3\sigma_4$. The linking number of the two strands in the resulting cable is then the exponent sum of the original 3-braid, so that the cables produced in this way will in this example be the (2,12) cables about C_1 and C_2 respectively. Addition of one extra σ_1 to each 6-braid will give 1-component knots K_1, K_2, the (2,13) cables about C_1, C_2. In table 1 the polynomials P_{K_1} and P_{K_2} are exhibited; they can be seen to differ considerably, as do the Jones polynomials of K_1 and K_2, while their Alexander polynomials, which satisfy a satellite formula, do not.

One further pair is given in table 2 of Birman's more general type. These pairs are given by $\beta_1 = \delta_1\delta_2\delta_3$, $\beta_2 = \delta_1\delta_3\delta_2$, where

$$\delta_1 = \sigma_1^{p_1}\sigma_2^{q_1}\sigma_1^{p_2}\sigma_2^{q_2}, \quad \delta_2 = \sigma_1^{p_1}\sigma_2^{q_2}, \quad \delta_3 = \sigma_1^{p_1-p_2}\sigma_2^{q_2-q_1}.$$

All other Birman pairs which we have tried can be distinguished by their 2-cable polynomials. This may be compared with Birman's difficulties in distinguishing the closed 3-braids using other methods.

Any of these examples will show also that the Burau polynomial is insufficient for calculating P, as noted earlier.

Symmetry and amphicheirality

The polynomial P_K of an amphicheiral knot K is unchanged when v is replaced by $-v^{-1}$. The knot 9_{42} has

$$P(v,z) = (2+z^2)v^{-2} - (3+4z^2+z^4) + (2+z^2)v^2,$$

Table 3

Braid representing the knot 9_{42}
*** -21332-1-32-12-34567 ***

	−2	0	2
0	2	−3	2
2	1	−4	1
4		−1	

Braid representing the (2, 0)-cable about 9_{42}
*** $(4354)^{-1}2132(6576)^{2}4354(2132)^{-1}(6576)^{-1}4354(2132)^{-1}4354(6576)^{-1}1^{-2}$ ***

The polynomial P for this cable

	−5	−3	−1	1	3	5	7	Jones polynomial
−1	4	−16	29	−29	16	−4		−1 (−21)
1	30	−109	159	−124	46	5	−7	1 (−19)
3	76	−274	341	−201	43	29	−14	−1 (−5)
5	85	−338	376	−159	16	27	−7	1 (−3)
7	45	−221	231	−65	2	9	−1	−1 (−1)
9	11	−78	79	−13		1		−1 (1)
11	1	−14	14	−1				1 (3)
13		−1	1					−1 (5)
								1 (19)
								−1 (21)

Alexander Polynomial = Conway polynomial = 0

which has this symmetry although 9_{42} is not amphicheiral. The (2, 0) cable about an amphicheiral knot will again be amphicheiral, so its polynomial will be symmetric. Calculation of the polynomial for the (2, 0) cable about 9_{42} is displayed in Table 3. This polynomial is not symmetric, so giving a proof that 9_{42} is not amphicheiral. Notice that the Jones polynomial in table 3 still exhibits symmetry, so that it does not detect the lack of amphicheirality, in this case, nor apparently does Kauffman's new polynomial F.

Braid index

It was shown in [**10**], and also in [**3**], that a lower bound for the braid index of K can be found from the polynomial P_K.

Explicitly, $n \geqslant \frac{1}{2}(e_{\max} - e_{\min}) + 1$, where n is the braid index of K, i.e. the smallest number of strings needed to present K as a closed braid, $e_{\max}$ and $e_{\min}$ are the largest and smallest degree respectively of the non-Alexander variable v in $P_K(v, z)$. This same inequality is shown in [**10**] to apply also where n is the number of Seifert circles arising from any diagram of K.

In a number of instances the braid index inequality can be shown to be strict, and the braid index calculated exactly, by applying the inequality to a cable about K. For example the (2, 7) cable K about a trefoil can be presented as the closure of the 4-braid $\sigma_1(\sigma_2\sigma_3\sigma_1\sigma_2)^3$, but it has $\frac{1}{2}(e_{\max} - e_{\min}) + 1 = 3$. This was noted by Franks and Williams [**4**], who asked whether its braid index was actually 3. Our calculations presented in Table 4 exclude this possibility. For if K has a presentation as a 3-braid, then every 2-cable about K has a presentation as a 6-braid, and so the polynomial

Table 4

Braid representing the (2, 27)-cable about the (2, 7)-cable about the trefoil
*** $2132(4354657621324354)^{3}1$ ***

The polynomial P for this knot

46	48	50	52	54	56	58	
893	−2935	3740	−2295	675	−79	2	0
24030	−66203	68350	−32340	6775	−485	4	2
310359	−713763	591303	−211414	30170	−1161	1	4
2503447	−4798408	3170667	−844269	79190	−1461		6
13881384	−22194447	11658704	−2284522	136147	−1068		8
55537145	−74293024	30971785	−4416455	161397	−468		10
165324695	−185800514	61398463	−6296288	135570	−121		12
374554353	−355168252	92877922	−6755584	81723	−17		14
657426753	−527978406	108944297	−5525984	35398	−1		16
906832669	−618538005	100234894	−3470613	10901			18
994140130	−576814659	72894389	−1676680	2325			20
873633502	−431235362	42078814	−620805	326			22
619135093	−259594689	19295646	−174377	27			24
355091052	−126031335	7007766	−36455	1			26
164964919	−49280982	2000681	−5489				28
61960207	−15447015	443071	−562				30
18717410	−3846817	74482	−35				32
4505528	−750139	9177	−1				34
851448	−111970	781					36
123451	−12342	41					38
13245	−946	1					40
990	−45						42
46	−1						44
1							46

Table 5

Braid representing a 2-cable about Conway's 11-crossing knot
*** $(4354)^{3}2132(6576)^{-1}(4354)^{-2}2132(4354)^{-1}2132(6576)^{-1}1$ ***

Braid representing a similar 2-cable about the Kinoshita-Teresaka knot
*** $(2132)^{3}(6576)^{2}4354(6576)^{-1}(2132)^{-2}4354(6576)^{-1}(2132)^{-1}(4354)^{-1}1$ ***

The polynomial P for both these knots

−2	0	2	4	6	8	10	12	
15	−97	233	−252	101	33	−43	11	0
146	−861	1917	−1926	646	341	−344	82	2
688	−3533	7068	−6430	1815	1251	−1115	256	4
1831	−8531	15171	−12175	2879	2352	−1982	455	6
2921	−13081	20828	−14371	2800	2547	−2115	471	8
2870	−13145	19014	−10997	1714	1656	−1389	277	10
1757	−8781	11703	−5514	656	651	−562	90	12
667	−3908	4850	−1790	151	151	−136	15	14
152	−1142	1330	−361	19	19	−18	1	16
19	−210	231	−41	1	1	−1		18
1	−22	23	−2					20
	−1	1						22

for each cable would satisfy $\frac{1}{2}(e_{max} - e_{min}) \leqslant 6$, as would also be the case if any diagram for K had just 3 Seifert circles.

The 8-braid used in Table 4 represents the $(2, 27)$ cable about K. The seven non-zero columns of coefficients show that $\frac{1}{2}(e_{max} - e_{min}) + 1 = 7$, so that at least seven strings are needed to present this cable. The knot K is then an example of the closure of a positive braid with braid index strictly larger than $\frac{1}{2}(e_{max} - e_{min}) + 1$.

Mutants

A knot K whose diagram is made up of tangles R and S as in figure 1 is converted into a *mutant* of K by replacing R with $\tau(R)$, where τ is the operation of rotation through π about one of three axes.

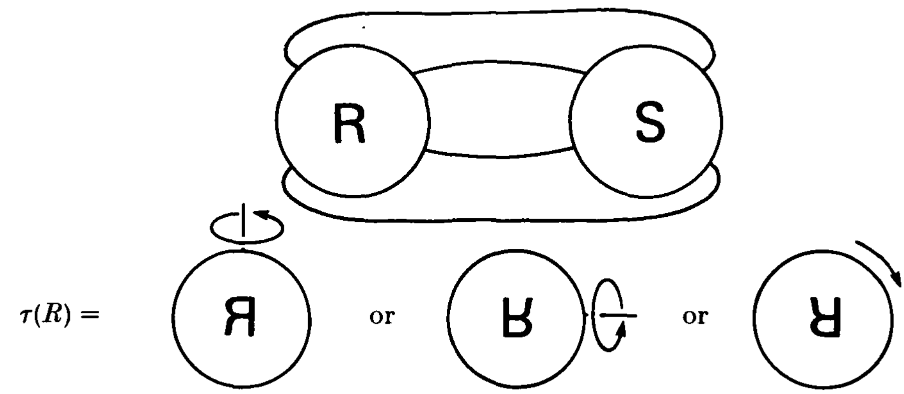


Fig. 1

Mutants have long been known to have the same Alexander polynomial, and more recently to have the same polynomial P [**9**]. Possibly the best known mutant pair are the inequivalent knots of Conway and Kinoshita-Terasaka which both have trivial Alexander polynomial. Although their 2-cables are not obviously mutants they do still share the same polynomial P shown in Table 5. This coincidence of polynomials has given us a measure of confidence in the accuracy of the computer calculations, as it would appear highly unlikely that the coefficients would all agree if there was an error in the algorithm or its implementation.

Other pairs of mutants within the range of our computations have been tried with similar results. A variety of mutants can be tackled using the fact that the 4-braids $w(\sigma_1, \sigma_2)\, v(\sigma_2, \sigma_3)$, $w(\sigma_1, \sigma_2)\, \sigma_2^{-1}\, v(\sigma_2, \sigma_3)\, \sigma_2$, $w(\sigma_1, \sigma_2)\, \mathrm{rev}\, v(\sigma_2, \sigma_3)$ and $w(\sigma_1, \sigma_2)\, \sigma_2^{-1}\, \mathrm{rev}\, v(\sigma_2, \sigma_3)\, \sigma_2$ close to mutants, where rev v is the braid v in reverse.

The result suggested by these calculations, that 2-cables of any pair of mutant knots have the same polynomial, has been subsequently proved by Lickorish and Lipson [**15**], and also by Przytycki and Traczyk.

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