How warming and steric sea level rise relate to cumulative carbon emissions

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[1] Surface warming and steric sea level rise over the global ocean nearly linearly increase with cumulative carbon emissions for an atmosphere-ocean equilibrium, reached many centuries after emissions cease. Surface warming increases with cumulative emissions with a proportionality factor, $\Delta T_{surface:2\times CO_2}/(I_B \ln 2)$, ranging from 0.8 to 1.9 K (1000 PgC)$^{-1}$ for surface air temperature, depending on the climate sensitivity $\Delta T_{surface:2\times CO_2}$ and the buffered carbon inventory I_B. Steric sea level rise similarly increases with cumulative emissions and depends on the climate sensitivity of the bulk ocean, ranging from 0.4 K to 2.7 K; a factor 0.4 ± 0.2 smaller than that for surface temperature based on diagnostics of two Earth System models. The implied steric sea level rise ranges from 0.7 m to 5 m for a cumulative emission of 5000 PgC, approached perhaps 500 years or more after emissions cease. **Citation:** Williams, R. G., P. Goodwin, A. Ridgwell, and P. L. Woodworth (2012), How warming and steric sea level rise relate to cumulative carbon emissions, *Geophys. Res. Lett.*, *39*, L19715, doi:10.1029/2012GL052771.

1. Introduction

[2] There is an ongoing challenge to understand the ocean warming and sea level rise due to enhanced radiative forcing from increasing atmospheric carbon dioxide. Recently coupled climate-carbon models have emphasized the importance of cumulative carbon emissions, rather than emission path, in determining peak warming [*Allen et al.*., 2009] and the climate response after emissions cease [*Matthews et al.*, 2009; *Zickfeld et al.*, 2009]. To understand the connection between ocean warming, steric sea level rise and cumulative carbon emissions, we develop an analytical theory for an atmosphere-ocean equilibrium, approached on timescales of many hundreds to a thousand years, drawing upon how the ocean thermally expands and how carbon dioxide is partitioned between the atmosphere and ocean. The analytical relations include the effects of increasing acidity and warming in reducing the effectiveness of the ocean in taking up carbon, and so enhancing the climate impact of carbon emissions.

[3] In order to understand the effect of carbon emissions on steric changes in sea level, we firstly set out the link between the surface heat flux and the rate of steric sea level rise for a global-mean ocean (Section 2); identify how long-term surface warming depends on cumulative carbon emissions, then explore the similar steric sea level response, diagnosing the climate sensitivity for the bulk ocean from two Earth System models, estimating the likely equilibrium timescale using scale analysis, and include the amplification from solubility feedback (Section 3); and finally discuss the wider implications (Section 4).

2. Rate of Steric Sea Level Rise From Ocean Warming

[4] The global-mean steric change in sea level, η, depends on the depth-integrated change in density over the global ocean,

$$\Delta\eta = \eta(t) - \eta(t_0) = -\frac{1}{\rho_0}\int_{-D}^{0} \overline{\Delta\rho(z)}^A dz, \tag{1}$$

where D is the maximum ocean depth and ρ_0 is a reference density, taken as 1026 kg m^{-3} from a global average over the upper 100 m, and the overbar represents a global average over the ocean area A, such that $\overline{\Delta\rho(z)}^A \equiv \frac{1}{A}\int_A \Delta\rho(z)dA$. The density change for the bulk ocean is estimated using a linear approximation for the equation of state, $\Delta\rho = -\alpha\rho_0\Delta T + \beta\rho_0\Delta S$, so that the steric sea level change is then related to the global, volume-weighted changes in ocean temperature, ΔT_{ocean}, and salinity, ΔS_{ocean},

$$\begin{aligned}\Delta\eta &= \int_{-D}^{0}\left(\overline{\alpha(z)}^A\overline{\Delta T(z)}^A - \overline{\beta(z)}^A\overline{\Delta S(z)}^A\right)dz \\ &= \overline{\alpha}^V D\Delta T_{ocean} - \overline{\beta}^V D\Delta S_{ocean},\end{aligned} \tag{2}$$

where $\alpha = -\frac{1}{\rho}\frac{\partial\rho}{\partial T}\big|_{S,P}$ is the thermal expansion coefficient (K^{-1}) and $\beta = \frac{1}{\rho}\frac{\partial\rho}{\partial S}\big|_{T,P}$ is the haline contraction coefficient (g kg^{-1})$^{-1}$, and the overbar represents a global average over the ocean volume V, such that $\overline{\alpha}^V \equiv \frac{1}{V}\int_V \alpha dV$, and $\Delta T_{ocean} = \frac{1}{D}\int_{-D}^{0}\overline{\Delta T(z)}^A dz$; this linearization neglects how $\overline{\alpha}^V$ and $\overline{\beta}^V$ covary with bulk property changes.

[5] Changes in steric sea level are affected by both thermal and haline changes on a basin scale, but are dominated by thermal changes when integrated over the global ocean [*Church et al.*, 2010]. Applying a heat balance for the global ocean allows the rate of ocean warming to be related to the surface heat flux, $\mathcal{H}$, averaged over the sea surface,

$$\frac{\partial T_{ocean}}{\partial t} = \frac{1}{\rho_0\overline{C_p}^V}\frac{\mathcal{H}}{D}, \tag{3}$$

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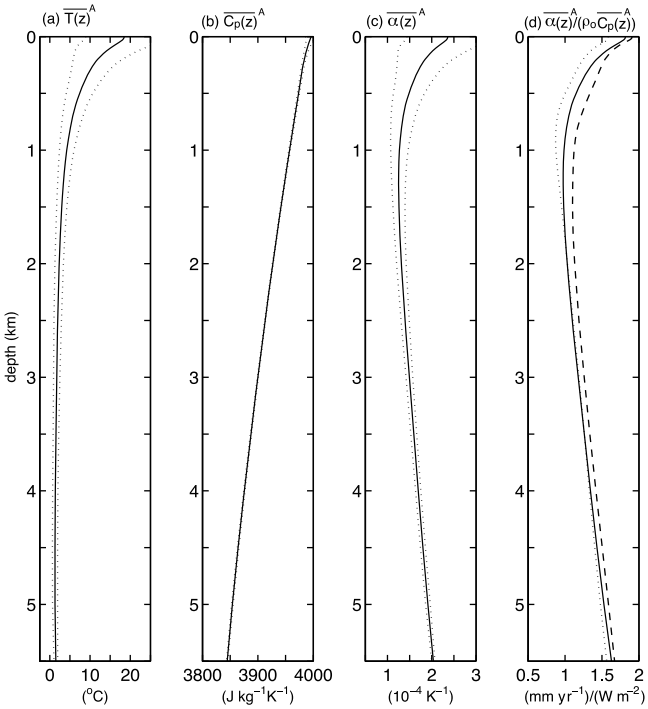

Figure 1. Global-mean, area-averaged profiles for (a) in situ temperature, $\overline{T(z)}^A$ (°C), (b) heat capacity, $\overline{C_p(z)}^A$ (J kg^{-1} K^{-1}), and (c) thermal expansion coefficient for density, $\overline{(\alpha,z)}^A$ (K^{-1}). The global mean (solid line) and one standard deviation from the mean (dotted lines) are shown based upon volumetric weighting of individual profiles for every one degree grid point of data over the globe (World Ocean Data 2001). In addition, (d) the factor $\overline{\alpha(z)}^A/\left(\rho_0\overline{C_p(z)}^A\right)$ in (4) to convert surface heat flux (W m^{-2}) to a rate of steric sea level rise (mm yr^{-1}) is illustrated as a mean profile over the global ocean (solid line), over the South Atlantic (dotted line) and the North Atlantic (dashed line); their depth mean values are 1.236, 1.175 and 1.345 mm yr^{-1} (W m^{-2})$^{-1}$ respectively.

where $\overline{C_p}^V$ is the heat capacity for the global-mean ocean. Combining (2) and (3) allows the rate of steric sea level rise to be related to the surface heat flux,

$$\frac{\partial\eta}{\partial t} = -\frac{1}{\rho_0}\int_{-D}^{0}\frac{\overline{\partial\rho}^A}{\partial t}\,dz \approx \frac{\overline{\alpha}^V}{\rho_0\overline{C_p}^V}\mathcal{H}. \tag{4}$$

[6] Our aim is to explore how surface warming and steric sea level rise are connected to cumulative carbon emissions, exploiting (2) for an atmosphere–ocean equilibrium approached after many hundreds to a thousand years. Prior to making this estimate, we consider the implications of the global-mean balance (4), so as to gain insight into the likely errors. The rate of steric sea level rise is affected by the response of the thermal expansion coefficient, α, and heat capacity, C_p, which are diagnosed from full-depth profiles of temperature and salinity using one-degree gridded, climatological data over the globe (Figure 1a, World Ocean Data 2001). C_p decreases in colder water and has a global mean, volume-weighted value of $\overline{C_p}^V = 3.910 \pm 0.001 \times 10^3$ J kg^{-1} K^{-1} (Figure 1b), while α increases in warmer waters and with pressure, and has a global-mean weighted value of $\overline{\alpha}^V = 1.572 \pm 0.147 \times 10^{-4}$K^{-1} (Figure 1c). The steric sea level rise is then obtained by multiplying the surface heat flux by a factor $\overline{\alpha}^V/\left(\overline{C_p}^V\rho_0\right) = 1.236 \pm 0.116$ mm yr^{-1} (W m^{-2})$^{-1}$ (Figure 1d, solid line).

[7] Assuming our global-mean relationship (4) and a surface heat flux of 0.53 W m^{-2} [*Church et al.*, 2011] (diagnosed from full depth changes in ocean heat content from 1972 to 2008), then implies a steric sea level rise of 0.66 ± 0.05 mm yr^{-1}. This global-mean estimate is less than the corresponding diagnostic of steric sea level rise of 0.80 ± 0.15 mm yr^{-1} [*Church et al.*, 2011]. While the estimates are almost consistent given their uncertainties, their mismatch suggests an error of 0.14 mm yr^{-1} or nearly 20% of the signal. This underestimation is likely to reflect the effect of applying a global mean and not taking into account the spatial pattern of the warming, which is more pronounced in the upper ocean and the North Atlantic. There is a greater expansion in warmer, upper waters, as reflected in the depth-mean conversion factor $\overline{\alpha}^V/\left(\overline{C_p}^V\rho_0\right)$ increasing from 1.175 to 1.345 mm yr^{-1}(W m^{-2})$^{-1}$ for the cooler South Atlantic to the warmer North Atlantic respectively (Figure 1d, dotted and dashed lines).

[8] Our aim is next to derive global-mean relationships between surface warming, steric sea level rise and cumulative carbon emissions.

3. Long-Term Effect of Carbon Emissions

[9] Atmospheric CO_2 is presently increasing due to the combined effect of carbon emissions and land use changes. Establishing the precise relationship between atmospheric CO_2 and these forcing functions is complicated due to the ocean uptake and cycling of carbon, as well as the terrestrial uptake. Carbon dioxide is transferred between the atmosphere and surface ocean with a local equilibration timescale of typically 1 year. Within the water column, there is a rapid chemical equilibrium with carbon exchanged between dissolved CO_2 and much larger carbonate and bicarbonate pools. This dissolved inorganic carbon, DIC, is biologically utilised within the sunlit, surface ocean and converted into organic carbon, and part of this organic carbon is exported to the dark interior and respired, increasing the concentration of DIC at depth. At the same time, more DIC is held in cooler, more soluble waters and is physically transported over the globe.

[10] Despite the complexity of many of these ocean processes, eventually an atmosphere-ocean equilibrium is approached, when further exchange of carbon between the atmosphere and global ocean ceases; see the box model illustration in Figure 2a. At this long-term equilibrium, atmospheric CO_2 varies exponentially with the cumulative carbon emissions [*Goodwin et al.*, 2007],

$$CO_2\left(t_{equilib}\right) = CO_2(t_0)\exp(\Delta I_{emission}/I_B), \tag{5}$$

where CO_2 is the atmospheric mixing ratio for carbon dioxide (ppmv), $t_{equilib}$ is the timescale for this equilibrium state to be approached, $\Delta I_{emission}$ is the cumulative carbon emission (PgC) and I_B is the buffered carbon inventory,

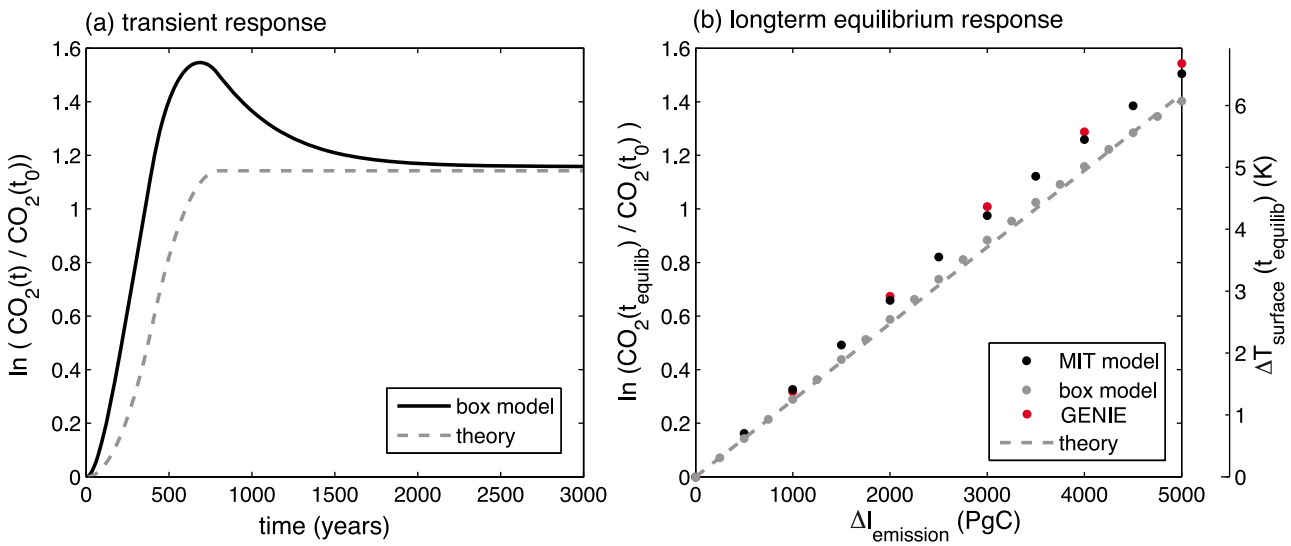

Figure 2. Atmospheric CO_2 response to carbon emissions: (a) transient response for an idealised box model and (b) final equilibrium response for atmospheric $CO_2(t_{equilib})$ versus cumulative carbon emissions, $\Delta I_{emission}$ (PgC) from a range of models; CO_2 displayed as logarithmic variation relative to the present day, $CO_2(t_0)$. In Figure 2a, a carbon emission pulse of 4000 PgC over 800 years is applied to an idealised box model (with a well mixed atmosphere, surface tropical, high latitude and deep ocean boxes) leading to atmospheric CO_2 increasing and, after emissions ceases, reducing slightly to a long-term equilibrium. Analytical projections from (5) (dashed line) are included in Figure 2a and compared in Figure 2b with the idealised box model (grey dots), an Earth System model (GENIE) (red dots) and a MIT ocean global circulation model (black dots) after 3000 years, each dot is a separate model integration. On the right-hand axis of Figure 2b, there is the implied change in surface temperature (7) and the slope represents the proportionality factor in (8). The spread in the model results are due to the choice of the buffered carbon inventory, I_B.

typically 3500 PgC, depending on the size of the atmosphere and ocean carbon inventories, and the ocean buffer factor.

[11] This exponential relationship for atmospheric CO_2 represents a positive feedback from ocean acidity: increasing CO_2 leads to increasing acidity, reducing the proportion of dissolved carbonate ions and increasing the proportion of dissolved CO_2, which then inhibits subsequent ocean uptake of CO_2. This ocean acidity feedback leads to a smaller and smaller fraction of emitted carbon in the atmosphere being progressively taken up by the ocean.

[12] This analytical prediction (5) for how atmospheric CO_2 varies with carbon emissions is tested by comparing with a suite of model experiments: an idealised 3-box model, an Earth System model (GENIE) [*Cao et al.*, 2009] and 10 separate experiments with the MIT ocean coupled primitive equation and carbon cycle model with cumulative carbon emissions progressively increasing to 4000 PgC, each model integrated to 3000 years (Figure 2b) [*Goodwin et al.*, 2007]. There is close agreement between the analytical theory and the model results; the slight differences are due to the buffered carbon inventory in each model, I_B is 3100 PgC in the MIT model, 3200 PgC in the GENIE model, 3410 PgC in the box model and 3500 PgC is the best estimate from the data [*Goodwin et al.*, 2007, 2009].

3.1. Long-Term Heating and Surface Warming

[13] The radiative heat flux at the sea surface increases logarithmically with increasing atmospheric CO_2 [*Myhre et al.*, 1998],

$$F(t) = a \ln\left(CO_2(t)/CO_2(t_0)\right), \tag{6}$$

where $a = 5.35$ W m^{-2} assuming an adjustment of only the upper atmosphere, the stratosphere. However, on timescales of several hundred years or more, the continual radiative heat input, $F(t)$, is expected to decline as a radiative adjustment of the troposphere occurs. The resulting heat input then leads to a surface warming, represented by

$$\Delta T_{surface}(t) = \Delta T_{surface:2\times CO_2} \frac{\ln(CO_2(t)/CO_2(t_0))}{\ln 2}, \tag{7}$$

where the climate sensitivity, $\Delta T_{surface:2\times CO_2}$, is the surface temperature increase for a doubling of atmospheric CO_2 and varies from 2 K to 4.5 K, with a mean of 3 K, from a range of climate models [*Knutti and Hegerl*, 2008].

[14] For a long-term equilibrium, $t_{equilib}$, the global surface temperature change is then linearly related to the cumulative carbon emissions by combining (5) and (7),

$$\Delta T_{surface}\left(t_{equilib}\right) = \left(\frac{\Delta T_{surface:2\times CO_2}}{I_B \ln 2}\right)\Delta I_{emission}. \tag{8}$$

This linear relationship with cumulative carbon emissions is also empirically found in climate-carbon model integrations [*Matthews et al*, 2009] with a proportionality factor diagnosed as 1.5 K (1000 PgC)$^{-1}$; also illustrated by the slope in Figure 2b, right axis.

[15] Our theory provides an analytical relation for the proportionality factor, $\Delta T_{surface:2\times CO_2}/(I_B \ln 2)$, which independently gives the same value as that empirically diagnosed assuming a climate sensitivity of 3.6 K in the UVic Earth System model [*Zickfeld et al.*, 2009] and a buffered carbon inventory, I_B, of 3500 PgC. Assuming a wider range for the climate sensitivity of 2 K to 4.5 K [*Knutti and Hegerl*,

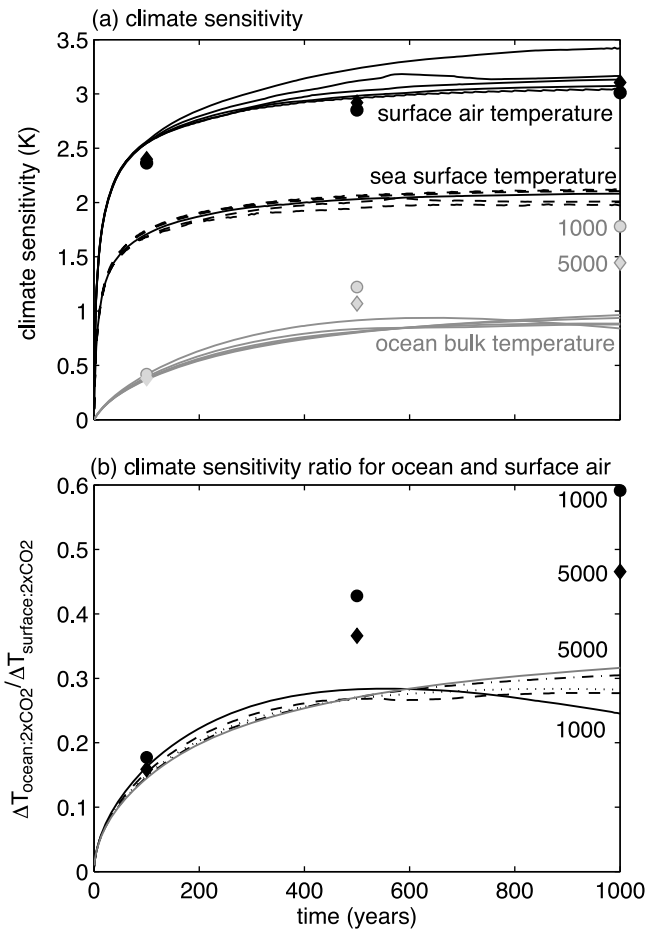

Figure 3. Diagnostics of the climate sensitivity with time for 1000 years with cumulative carbon emissions (increased every 1000 PgC from 1000 to 5000 PgC) from the GENIE Earth System model and selected estimates (1000 PgC is circles, 5000 PgC is diamonds) from a pulse experiment using the UVic Earth System model: (a) climate sensitivity for surface air temperature (black line and symbols), sea surface temperature (dashed line) and ocean bulk temperature (grey line and symbols) and (b) ratio of the climate sensitivity for the bulk ocean and surface air temperature, $\Delta T_{ocean:2\times CO_2}/\Delta T_{surface:2\times CO_2}$, for each separate integration (1000 PgC is solid black, 2000 PgC is dashed, 3000 PgC is dotted, 4000 PgC is dot-dashed, 5000 PgC is solid grey for GENIE).

2008], then implies the proportionality factor varies from 0.8 to 1.9 K (1000 PgC)$^{-1}$, which is comparable to empirical diagnostics of the proportionality factor, 1 to 2.1 K (1000 PgC)$^{-1}$, from a range of C^{4}MIP models [*Zickfeld et al.*, 2012].

[16] The surface warming of the ocean can be similarly related to cumulative emissions using (8) with a climate sensitivity chosen for the surface ocean (Figure 3a).

3.2. Sea Level Change and Climate Sensitivity for the Ocean

[17] The long-term steric sea level change depends on the interior density changes, rather than solely on surface temperature changes, such that for a global mean (2) relates

$$\Delta\eta = -\frac{1}{\rho_0}\int_{-D}^{0}\overline{\Delta\rho(z)}^{A}dz \approx \overline{\alpha}^{V}D\Delta T_{ocean},$$

where the changes in salinity are again ignored. In order to assess the global sea level response, a climate sensitivity for the bulk ocean, $\Delta T_{ocean:2\times CO_2}$, is required,

$$\Delta T_{ocean}(t) = \Delta T_{ocean:2\times CO_2}\frac{\ln(CO_2(t)/CO_2(t_0))}{\ln 2}. \qquad (9)$$

The warming of the ocean interior does partly track the surface warming due to the ventilation process, transferring surface properties into the ocean interior, but becomes more complicated due to how ice affects the temperature of bottom waters.

[18] To reveal how these climate sensitivities, $\Delta T_{surface:2\times CO_2}$ and $\Delta T_{ocean:2\times CO_2}$, compare, two Earth System models are diagnosed for carbon emissions from 1000 to 5000 PgC. In GENIE [*Cao et al.*, 2009], the climate sensitivity is typically 3 K for surface air temperature, reducing to 2 K for sea surface temperature and nearly 1 K for the ocean bulk temperature (Figure 3a, lines). In the UVic Earth System model for a pulsed emission [*Archer et al.*, 2009], the climate sensitivity after 1000 years is typically 3.1 K for surface air temperature and 1.4 K to 1.8 K for ocean bulk temperature (Figure 3a, symbols). The climate sensitivity for the bulk ocean does vary temporally with the circulation, particularly with the Atlantic meridional overturning cell, and the presence of ice.

[19] The ratio of the climate sensitivity for the bulk ocean and surface air temperature, $\Delta T_{ocean:2\times CO_2}/\Delta T_{surface:2\times CO_2}$, is 0.25 to 0.3 for GENIE and 0.4 to 0.6 for UVic for a timescale of 500 to 1000 years (Figure 3b); hence, we assume that this ratio is typically 0.4 ± 0.2. Assuming the climate sensitivity for surface temperature ranges from 2 K to 4.5 K based on a wide range of climate models [*Knutti and Hegerl*, 2008], then the above ratio implies that the climate sensitivity for the bulk ocean ranges from 0.4 K to 2.7 K.

[20] Combining (2), (5) and (9), then gives a projection for the steric sea level change with a linear dependence on cumulative carbon emissions for a long term equilibrium,

$$\Delta\eta(t_{equilib}) \approx \left(\frac{\overline{\alpha}^{V}D\Delta T_{ocean:2\times CO_2}}{I_B\ln 2}\right)\Delta I_{emission}. \qquad (10)$$

The proportionality factor, $\overline{\alpha}^{V}D\Delta T_{ocean:2\times CO_2}/(I_B\ln 2)$, is similar to that in (8), but also depends on the thermal expansion coefficient, ocean depth and climate sensitivity for the bulk ocean. A cumulative emission of 1000 PgC implies a sea level change ranging from 0.1 m to 0.9 m depending on the choice of $\Delta T_{ocean:2\times CO_2}$ (Figure 4, dashed lines).

3.3. Timescale for an Atmosphere-ocean Equilibrium State to Be Reached

[21] In order to estimate when the atmosphere-ocean equilibrium state is approached, a scale analysis is applied to the heat equation for the global ocean (3) to obtain a timescale, $\mathcal{T}$,

$$\mathcal{T} \sim \frac{\Delta T_{ocean}}{\partial T_{ocean}/\partial t} = \frac{\rho_0\overline{C_p}^{V}D\Delta T_{ocean}}{\mathcal{H}}. \qquad (11)$$

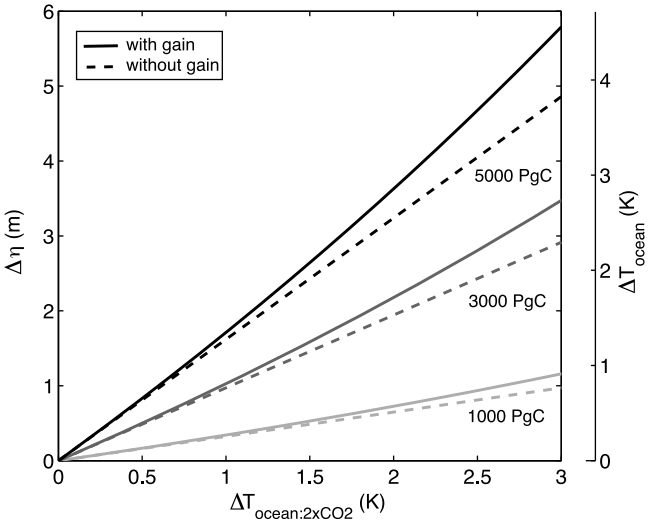

Figure 4. Long-term change in steric sea level, $\Delta\eta$ (m) (left axis) and ocean bulk temperature, ΔT_{ocean} (right axis) versus climate sensitivity for the bulk ocean, $\Delta T_{ocean:2\times CO_2}$ (K), for a range of cumulative carbon emissions based on our theory: 1000 PgC (light grey), 3000 PgC (mid grey) and 5000 PgC (black line) with a larger response for increasing cumulative emissions. There is a linear response when considering the effect of carbon emissions in isolation (10) (dashed lines), but a slight amplification with increasing emissions when including the warming and solubility feedback (13) (solid lines).

If the temperature anomaly ΔT_{ocean} is replaced by the climate sensitivity (9) and increase in radiative heat flux $\mathcal{H}$ by the dependence on atmospheric CO_2 (6), $\mathcal{H} \sim fF\left(t_{equilib}\right)/2$, reduced by a factor 2 to represent an average over the adjustment and a factor $f = 0.93$ to account for not all the heat going into the ocean [*Church et al.*, 2011], then (11) becomes

$$\mathcal{T} \sim \frac{2\rho_0 \overline{C_p}^V D\Delta T_{ocean:2\times CO_2}}{fa\ln 2}. \tag{12}$$

This scaling suggests that the air-sea equilibrium is approached after carbon emissions cease on a timescale of the order of 500 years (using $\Delta T_{ocean:2\times CO_2} = 1.4$ K and $D \sim 5000$ m). As the ocean is treated as a well mixed box, this scaling represents an underestimate as the slowness of the deep circulation is not taken into account. In comparison, climate model experiments reveal the surface equilibrating to a quadrupling of atmospheric CO_2 after 1200 years and the deep ocean after 5000 years [*Li et al.*, 2012].

3.4. Amplification of Warming and Steric Sea Level Rise From Climate Feedback

[22] The relationship for surface warming (8) and steric sea level rise (10) take into account the positive feedback from ocean acidity due to increasing carbon emission. There is an additional weak positive feedback due to how warming reduces ocean solubility and further increases atmospheric CO_2, which is expressed in terms of the initial perturbation from carbon emissions multiplied by a gain factor, so that for sea level (10) becomes

$$\Delta\eta\left(t_{equilib}\right) \approx \underbrace{\left(\frac{\overline{\alpha}^V D\Delta T_{ocean:2\times CO_2}\Delta I_{emission}}{I_B \ln 2}\right)}_{perturbation} \cdot \underbrace{\left(1 - \frac{\partial \ln|CO_2|}{\partial T}\frac{\partial T}{\partial \ln|CO_2|}\right)^{-1}}_{gain}, \tag{13}$$

where the gain represents how more warming leads to reduced solubility, in turn leading to more atmospheric CO_2 and so enhances the original warming. The terms contributing to the gain depend on how warming affects the solubility of CO_2 [*Goodwin and Lenton*, 2009], $\partial \ln |CO_2|/\partial T = -(V/I_B)\ \partial C_{sat}/\partial T|_{CO_2}$, and the climate sensitivity for the ocean (9), such that $gain = \left(1 + \frac{\partial C_{sat}}{\partial T}\Big|_{CO_2}\frac{V\Delta T_{ocean:2\times CO_2}}{I_B \ln 2}\right)^{-1}$, where $\frac{\partial C_{sat}}{\partial T}\Big|_{CO_2} \sim 0.1$ gC m^{-3} K^{-1} is the change in the saturated carbon store with temperature and $V = 1.3 \times 10^{18}$ m^3 is the ocean volume. The gain varies from 1.02 to 1.17 for a climate sensitivity for the bulk ocean from 0.4 K to 2.7 K, which then slightly enhances the surface warming of the atmosphere and ocean, and steric sea level change (13) (Figure 4, solid line).

4. Conclusions

[23] The climate response to carbon emissions is usually assessed in terms of coupled climate-carbon models. Here we provide analytical relations to understand their long-term projections. Coupled climate-carbon models find that the response of surface climate variables, such as temperature, after emissions cease is nearly linearly related to cumulative carbon emissions [*Matthews et al*, 2009; *Zickfeld et al.*, 2009]. Our analytical theory suggests that the surface warming is connected to cumulative emissions by a proportionality factor, $\Delta T_{surface:2\times CO_2}/(I_B \ln 2)$, depending on the climate sensitivity for surface air temperature, $\Delta T_{surface:2\times CO_2}$, and the buffered carbon inventory, I_B. This nearly linear relationship is a consequence of how surface warming varies with the logarithm of atmospheric CO_2, while long-term atmospheric CO_2 varies exponentially with cumulative emissions due to an ocean acidity feedback [*Goodwin et al.*, 2009]. Surface warming of the ocean can be similarly defined (8) using a climate sensitivity for the surface ocean.

[24] Our analytical relations likewise reveal steric sea level rise nearly linearly increasing with cumulative carbon emissions, but the proportionality factor, $\overline{\alpha}^V D\Delta T_{ocean:2\times CO_2}/(I_B \ln 2)$, depends on the response of the ocean interior, involving the climate sensitivity for the bulk ocean, $\Delta T_{ocean:2\times CO_2}$, the thermal expansion coefficient, $\overline{\alpha}^V$, and ocean depth, D. A more complicated response is found in climate models with ocean heat content and steric sea level rising on a centennial timescale after emissions cease [*Zickfeld et al.*, 2012].

[25] Our theoretical relations are appropriate for a long-term climate when an air-sea equilibrium is approached

after emissions cease, reached on a timescale of the order of 500 years, perhaps delayed further by a slower adjustment of the deep ocean [*Li et al.*, 2012].

[26] In applying these analytical relationships, there are large uncertainties in using a global mean that ignores the spatial warming pattern and in the climate sensitivity of the bulk ocean, which depends on how the ocean is ventilated and the importance of surface ice.

[27] Conventional fossil fuel reserves are estimated to reach perhaps 5000 PgC [*Rogner*, 1997], albeit with large uncertainties in the recoverable fuel. Assuming that 5000 PgC is emitted to the atmosphere without any compensating carbon capture, then the theoretical relations suggest that the long-term steric sea level rise ranges from 0.7 m to 5 m (using climate sensitivities for the bulk ocean of 0.4 K to 2.7 K); the upper values are more appropriate whenever the ocean interior warming is close to that of the sea surface. This steric response is significant, larger than any sea level rise of up to 0.5 m from the mass addition from glaciers and ice caps [*Church et al.*, 2001, Table 11.3] and comparable to the input of several metres from substantial melting of the Greenland and Antarctic ice sheets.

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