MATH348. Harmonic Analysis. Problems 5.

Work is due in on *Friday 5th November*. I shall be away all day on Tuesday 2nd November, so there will be no office hours on that day. I shall be available at the usual times (11-1) on Monday and for part of the afternoon also, but I have to arrange two tutorials to Monday so am not yet sure which times. So I suggest having additional office hours on WEdnesday 3rd November, say 9-10 and 11-12. This is the reason for the later hand-in day, just for this week.

1. Find the Fourier transform $\hat{f}(\xi)$ of f, where a) for some $a > 0$, $f(x) = e^{-ax}$ for $x > 0$ and $= 0$ otherwise,

b) $f(x) = x$ for $0 \leq x \leq 1$ and $= 0$ otherwise,

c) $f(x) = xe^{-|x|}$.

2. Compute $\hat{f}(\xi)$ where

$$f(x) = \frac{1}{2 + 2x + x^2}.$$

In the case $\xi \geq 0$ you might find it helpful to consider the contour integral of $e^{-i\xi z}/(2 + 2z + z^2)$ round a half disc in the lower half plane. To get the formula for all ξ you may find it helpful to show that, as $f(x)$ is real for real x,

$$\hat{f}(-\xi) = \overline{\hat{f}(\xi)}$$

3. Find $\hat{f}(\xi)$ where

$$f(x) = \frac{1}{(1 + x^2)^2}.$$

you can use

$$\overline{\hat{f}(\xi)} = \hat{f}(-\xi).$$

4. Show that the function $1/(1 + ix)$ on $\mathbf{R}$ is not integrable. However, compute

$$I(\xi) = \lim_{\Delta \to \infty} \int_{-\Delta}^{\Delta} \frac{e^{-i\xi x} dx}{1 + ix}$$

by considering separately the cases $\xi = 0$, when you should show that

$$I(\xi) = \int_{-\infty}^{\infty} \frac{dx}{1 + x^2} = \pi,$$

and $\xi > 0$ and $\xi < 0$, by considering integrals of $e^{-i\xi z}/(1 + iz)$ round half-discs in the lower and upper half-planes respectively. You may assume that the integrals on the curved parts of the contours $\to 0$ as $\Delta \to \infty$. For $\xi > 0$ you should obtain that $I(\xi) = 0$.