MATH 342 Problem Sheet 2: Arithmetic

Due Monday 11th February

1. Find all the divisors of 1008. You may leave them in factorized form. Why should there be 30 divisors?

2.

a) Show that every integer can be written in the form $5a + 7b$ for a, $b \in \mathbb{Z}$.
Hint: Find a_1 and $b_1 \in \mathbb{Z}$ such that $5a_1 + 7b_1 = 1$

b) Show that every integer $n \geq 24$ can be written in the form $5a + 7b$ for a, $b \in \mathbb{N}$.
Hint: First do it for integers n with $24 \leq n \leq 28$.

c) Show that 35 cannot be written in the form $5a + 7b$ for a, $b \in \mathbb{Z}_+$, but 40 can be.

3. Find all integer solutions x to the following equations. In each case you may express the solution in the form $x \equiv y \mod p$ for an appropriate y and p

a) $x + 2 \equiv 1 \mod 4$

b) $3x \equiv 2 \mod 5$

c) $x^2 \equiv 1 \mod 3$

d) $x^3 \equiv 1 \mod 5$

e) $183x \equiv 1 \mod 257$. For this one you might like to use the Euclidean algorithm to find a and $b \in \mathbb{Z}$ such that $183a + 257b = 1$: if these exist. (They do, because 257 is prime.)

4. For any non-zero integers a, b and c,

$$\gcd(a, b) = \gcd(a + bc, b)\ .$$

Use this property to show that

$$\gcd(6n + 1, 6n - 3) = 1$$

and

$$\gcd(5n + 3, 3n + 2) = 1$$

for any integer n.

5. A key property of the integers (deduced from the Euclidean property) is that: if $gcd(a, b) = 1$ and $a \mid bc$, then $a \mid c$. Use this property to prove that: if $a \in \mathbb{Z}_+$ is prime and $b_i \in \mathbb{Z}_+$ for $1 \leq i \leq n$ and $a \mid \prod_{i=1}^{n} b_i$, then $a \mid b_i$ for at least one b_i.

I will collect solutions at the lecture on Monday 11th February. Any solutions which are not handed in then, or by 5pm that day in the folder outside room 516 will not be marked.