MATH 342 Problem Sheet 10: Quadratic reciprocity

Due Monday 29th April

1. Use the standard trick to switch "numerators" and "denominators" in Legendre symbols in order to compute

$$\left(\frac{3}{991}\right), \left(\frac{12}{991}\right), \left(\frac{5}{991}\right), \left(\frac{-10}{991}\right), \left(\frac{891}{991}\right).$$

You may assume that 991 is prime.

2. Compute

a) $\left(\frac{31}{97}\right)$;

b) $\left(\frac{53}{271}\right)$;

c) $\left(\frac{351}{787}\right)$.

Once again, you may assume that 97, 271 and 787 are prime.

3. Let p be an odd prime coprime to 5. Show that $\left(\frac{5}{p}\right) = 1$ if and only if $p \equiv \pm 1 \pmod 5$.

4. Show that if p is an odd prime coprime to 7, then $\left(\frac{7}{p}\right) = 1$ if and only if $p \equiv \pm 1, \pm 3$ or $\pm 9 \pmod{28}$.

Hint: If p is an odd prime, determine which values can p take mod 28, and consider each of these values in turn. Note that if we know p mod 28 then we know p mod 4, and hence we know whether $(p-1)/2$ is odd or even.

5. Show that there are infinitely many primes which are ± 1 mod 5.

Hint: Suppose there are only finitely many such primes, and let these primes be q_i for $1 \leq i \leq n$. Let

$$N = \prod_{i=1}^{n} q_i$$

and let p be any prime dividing $N^2 - 5$. Show that $p \equiv \pm 1$ mod 5, using question 3 or otherwise, and that $p \neq q_i$ for any $1 \leq i \leq n$.

I will collect solutions at the lecture on Monday 29th April. Any solutions which are not handed in then, or by 5pm that day in the folder outside room 516 will not be marked.