parts when at a distance do not act with any force on one another, the resistance is as the square of the velocity, accurately. Let there be, therefore, three mediums A, B, C, consisting of similar and equal parts regularly disposed at equal distances. Let the parts of the mediums A and B recede from each other with forces that are among themselves as T and V; and let the parts of the medium C be entirely destitute of any such forces. And if four equal bodies D, E, F, G move in these mediums, the two first D and E in the two first A and B, and the other two F and G in the third C; and if the velocity of the body D be to the velocity of the body E, and the velocity of the body F to the velocity of the body G, as the square root of the ratio of the force T to the force V; then the resistance of the body D to the resistance of the body E, and the resistance of the body F to the resistance of the body G, will be as the square of the velocities; and therefore the resistance of the body D will be to the resistance of the body F as the resistance of the body E to the resistance of the body G. Let the bodies D and F be equally swift, as also the bodies E and G; and, augmenting the velocities of the bodies D and F in any ratio, and diminishing the forces of the particles of the medium B as the square of the same ratio, the medium B will approach to the form and condition of the medium C at pleasure; and therefore the resistances of the equal and equally swift bodies E and G in these mediums will continually approach to equality, so that their difference will at last become less than any given. Therefore since the resistances of the bodies D and F are to each other as the resistances of the bodies E and G, those will also in like manner approach to the ratio of equality. Therefore the bodies D and F, when they move with very great swiftness, meet with resistances very nearly equal; and therefore since the resistance of the body F is in a squared ratio of the velocity, the resistance of the body D will be nearly in the same ratio.

COR. III. Hence the resistance of a body moving very swiftly in an elastic fluid is almost the same as if the parts of the fluid were destitute of their centrifugal forces, and did not fly from each other; provided only that the elasticity of the fluid arise from the centrifugal forces of the particles, and the velocity be so great as not to allow the particles time enough to act.

COR. IV. Since the resistances of similar and equally swift bodies, in a medium whose distant parts do not fly from each other, are as the squares of the diameters, therefore the resistances made to bodies moving with very great and equal velocities in an elastic fluid will be as the squares of the diameters, nearly.

COR. V. And since similar, equal, and equally swift bodies, moving through mediums of the same density, whose particles do not fly from each other, will strike against an equal quantity of matter in equal times, whether the particles of which the medium consists be more and smaller, or fewer and greater, and therefore impress on that matter an equal quantity of motion, and in return (by the third Law of Motion) suffer an equal reaction from the same, that is, are equally resisted; it is manifest, also, that in elastic fluids of the same density, when the bodies move with extreme swiftness, their resistances are nearly equal, whether the fluids consist of gross parts, or of parts ever so subtile. For the resistance of projectiles moving with exceedingly great celerities is not much diminished by the subtilty of the medium.

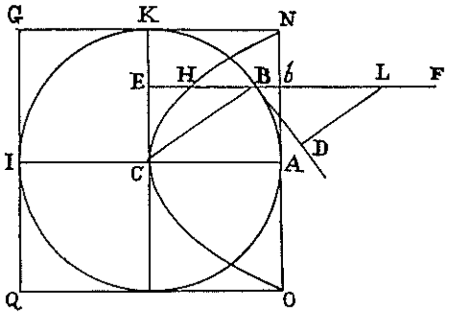
COR. VI. All these things are so in fluids whose elastic force takes its rise from the centrifugal forces of the particles. But if that force arise from some other cause, as from the expansion of the particles after the manner of wool, or the boughs of trees, or any other cause, by which the particles are hindered from moving freely among themselves, the resistance, by reason of the lesser fluidity of the medium, will be greater than in the Corollaries above.

PROPOSITION XXXIV. THEOREM XXVIII

If in a rare medium, consisting of equal particles freely disposed at equal distances from each other, a globe and a cylinder described on equal diameters move with equal velocities in the direction of the axis of the cylinder, the resistance of the globe will be but half as great as that of the cylinder.

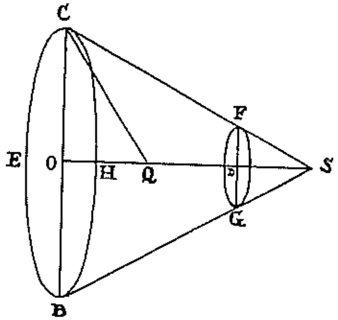
For since the action of the medium upon the body is the same (by Cor. v of the Laws) whether the body move in a quiescent medium, or whether the particles of the medium impinge with the same velocity upon the quiescent body, let us consider the body as if it were quiescent, and see with what force it would be impelled by the moving medium. Let, therefore, ABKI represent a spherical body described from the centre C with the semi-diameter CA, and let the particles of the medium impinge with a given velocity upon that spherical body in the directions of right lines parallel to

AC; and let FB be one of those right lines. In FB take LB equal to the semidiameter CB, and draw BD touching the sphere in B. Upon KC and BD let fall the perpendiculars BE, LD; and the force with which a particle of the medium, impinging on the globe obliquely in the direction FB, would strike the globe in B, will be to the force with which the same particle, meeting the cylinder ONGQ described about the globe with the axis ACI, would strike it perpendicularly in b, as LD is to LB, or BE to BC. Again; the efficacy of this force to move the globe, according to the direction of its incidence FB or AC, is to the efficacy of the same to move the globe, according to the direction of its determination, that is, in the direction of the right line BC in which it impels the globe directly, as BE to BC. And, joining these ratios, the efficacy of a particle, falling upon the globe obliquely in the direction of the right line FB, to move the globe in the direction of its incidence, is to the efficacy of the same particle falling in the same line perpendicularly on the cylinder, to move it in the same direction, as BE^2 to BC^2. Therefore if in bE, which is perpendicular to the circular base of the cylinder NAO, and equal to the radius AC, we take bH equal to $\frac{BE^2}{CB}$; then bH will be to bE as the effect of the particle upon the globe to the effect of the particle upon the cylinder. And therefore the solid which is formed by all the right lines bH will be to the solid formed by all the right lines bE as the effect of all the particles upon the globe to the effect of all the particles upon the cylinder. But the former of these solids is a paraboloid whose vertex is C, its axis CA, and latus rectum CA, and the latter solid is a cylinder circumscribing the paraboloid; and it is known that a paraboloid is half its circumscribed cylinder. Therefore the whole force of the medium upon the globe is half the entire force of the same upon the cylinder. And therefore if the particles of the medium are at rest, and the cylinder and globe move with equal velocities, the resistance of the globe will be half the resistance of the cylinder. Q.E.D.

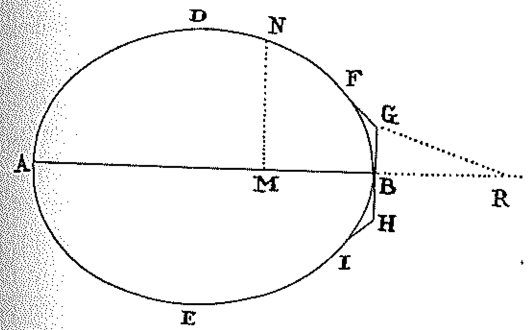


SCHOLIUM[1]

By the same method other figures may be compared together as to their resistance; and those may be found which are most apt to continue their motions in resisting mediums. As if upon the circular base CEBH from the centre O, with the radius OC, and the altitude OD, one would construct a frustum CBGF of a cone, which should meet with less resistance than any other frustum constructed with the same base and altitude, and going forwards towards D in the direction of its axis: bisect the altitude OD in Q, and produce OQ to S, so that QS may be equal to QC, and S will be the vertex of the cone whose frustum is sought.



Incidentally, since the angle CSB is always acute, it follows from the above that, if the solid ADBE be generated by the convolution of an elliptical or oval figure ADBE about its axis AB, and the generating figure be touched by three right lines FG, GH, HI, in the points F, B, and I, so that GH shall be perpendicular to the axis in the point of contact B, and FG, HI may be inclined to GH in the angles FGB, BHI of 135 degrees: the solid arising from the convolution of the figure ADFGHIE about the same axis AB will be less resisted than the former solid, provided that both move forwards in the direction of their axis AB, and that the extremity B of each go foremost. This Proposition I conceive may be of use in the building of ships.



If the figure DNFG be such a curve, that if, from any point thereof, as N, the perpendicular NM be let fall on the axis AB, and from the given point G there be drawn the right line GR parallel to a right line touching the

[1 Appendix, Note 35.]