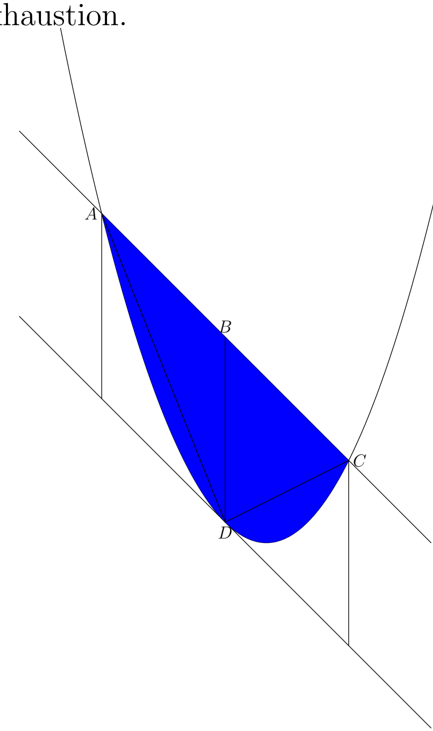
MATH302 History of Mathematics: Analysis and Calulus Assignment 1
Due on *Tuesday 28th February*

This assignment counts 5% towards the final assessment, and is due in by 5 p.m. on *Tuesday 28th February*. My office hours are Mondays 9-10 p.m. and Fridays 2-3 p.m. I am happy to discuss assignments at other times as convenient. Work can be given in, either to my box for handing in work outside the outer office 516 – or to me directly. Note that there are no MATH302 lectures in the week 27 February. There is a session with a librarian in CTC3 Chadwick on Monday 27 February, which I will visit, so anyone who is ready to hand in work at that time can hand it to me then. There is nothing arranged for Tuesday 28 February. The lecture slot at 12 p.m. on Friday 2 March will be an organisational meeting for presentations, in Room 106 as usual.

This assignment concerns Archimedes' proof that the area between a parabola $y = x^2$ and a line segment joining two points $A = (a, a^2)$ and $C = (c, c^2)$ on a parabola is four thirds of the area of the triangle ADC where D is the point on the parabola which is vertically below the midpoint B of AC Three proofs by Archimedes are mentioned in Kline's "Mathematical Thought from Ancient to Modern Times" (pp110-114). This question is based on Archimedes' third proof in Kline, which illustrates Archimedes' general technique of calculation of areas by the method of exhaustion.

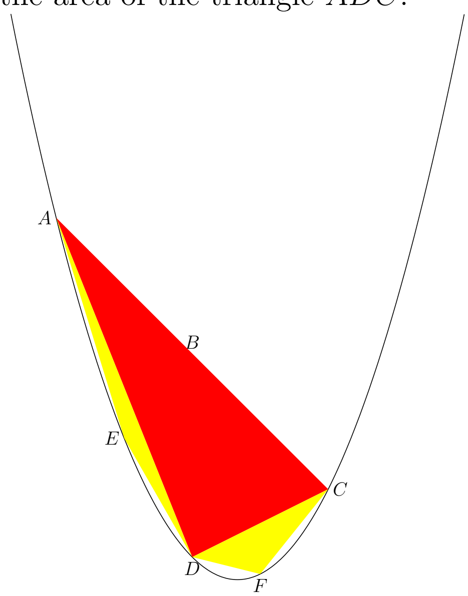


You are asked to prove the following for a particular choice of $A = (a, a^2)$ and $C = (c, c^2)$ which you are given. For your choices see the Individual Assignment Sheet.

1. For your choice of A and C calculate the areas of the triangles ABD and BCD and show that these are equal.

2. Show that the parabola is above the line through D which is parallel to AC, and that the section of the parabola between A and C is below the chord AC. (This may be obvious from the sketch but you should prove this by writing down the equations of the two lines and using the equation of the parabola.) Find the area of the parallelogram bounded by this line and the chord AC and vertical lines through A and C.

3. Use integration to show that the area between AC and the parabola is four thirds of the area of the triangle ADC.



4. Now let E and F be points on the parabola which vertically below the midpoints of AD and DC. Show that the area of each of the triangles AED and DFC is one eighth of the area of ACD.

5. Now consider A and C to be general points (a, a^2) and (c, c^2) on the parabola and define B and D as before. Show that the area of triangle ACD is proportional to $(c - a)^3$. hence show that in general the area of each of triangles AED and DFC is one eighth of the area of triangle ACD.