Recall that Kline lists four main areas which led to the introduction of calculus:

- formulae for distance, given speed or acceleration;
- tangents of curves;
- finding maxima and minima;
- finding lengths (and areas and volumes).

- The development of modern calculus started in the seventeenth century.
- A great many people were involved in doing calculations using the sort of arguments which developed into what we recognise as calculus.
- The motivation came from
  - mechanics (planetary motion, projectiles, pendulums),
  - optics;
  - sometimes more frivolous reasons e.g. the shape of wine barrels (Kepler)

Some names

- Galileo (1564-1642)
- Kepler (1571-1630)
- St Vincent (1584-1667)
- Descartes (1596-1650)
- Cavalieri (1598-1647)
- Fermat (1601-65)
- Roberval (1602-75)
- Huygens (1629-1695)

Some British names

In Britain there were:

- Isaac Barrow (1630-1677)(Isaac Newton's predecessor at Cambridge)
- John Wallis (1616-1703)
- James Gregory (1638-1675)
- Christopher Wren (1632-1723)

Galileo (1564-1642)

- He formulated the laws of mechanics in mathematical terms
- (He recognised distance as the area under velocity)
- At considerable personal cost. He was twice called before the Inquisition, and the second time (in 1633) was forced to recant his theories
- His last great work, the Dialogue concerning the Two Sciences, was published in 1638 in Holland, after being sent there secretly. This work was very influential.
- He was practically gifted, and built one of the first telescopes.
- He also solved some problems in the Calculus of Variations ... not always correctly..

Cavalieri (1598-1647)

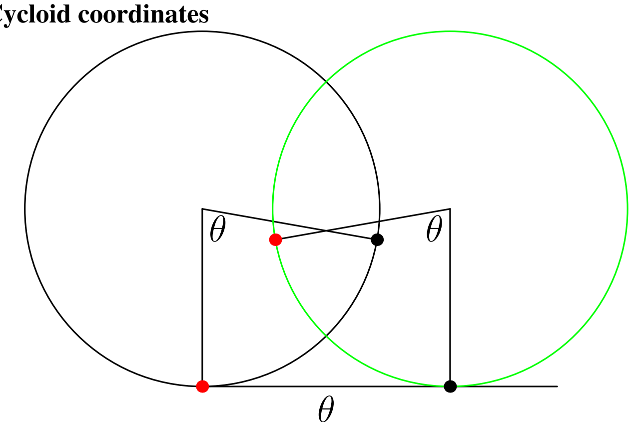
- He was a member of the Jesuati religious order from boyhood.
- He originally developed an interest in mathematics by reading Euclid.
- He was taught by Castelli, a lecturer at Pisa, and sometimes took his lectures.
- He regarded himself as a disciple of Galileo.
- He created the *theory of indivisibles*.
- In 1629 he was appointed to the chair of mathematics at the University of Bologna.

Roberval (1602-75)

- He was born Gilles Personne, but later took the name of his village (Roberval)
- His parents were farmers, and he was born in a field while his mother was bringing in the harvest.
- He was taught mathematics by the parish priest, a chaplain to the queen, who also taught him Latin and Greek.
- As a young man he earned his living teaching mathematics, and travelled widely, meeting many mathematicians, including Fermat.
- In 1634 he won the competition for the Ramus professorship in Paris, for which he had to compete for reappointment every three years.
- Possibly because of the need to keep his methods secret for the needs of competition, he published very little during his lifetime.
- He had a reputation as a miser, owning no property in Paris, but later buying a farm to the northwest.

- One of his best known works is *Traité des indivisibles*.
- The *theory of indivisibles* whether of Cavalieri or Roberval uses the idea of a plane area being made up of vertical or horizontal lines.
- If two areas are made up of sets of lines such that the corresponding lines are of equal length, then the areas are equal.
- As an example Roberval showed that the area under a cycloid arch traced by a disc of radius r, up to the highest point, is $\frac{3\pi}{2}r^2$.

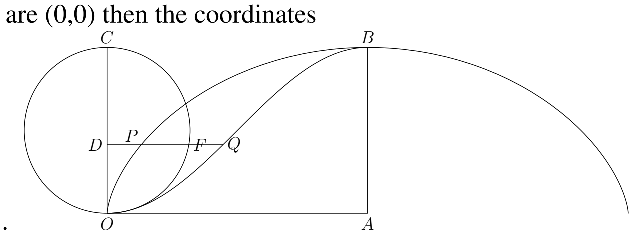
Cycloid coordinates



We will consider circles of radius 1.

The red and black points on the black circle roll to the red and black points on the green circle

If the coordinates of the red point on the black circle are (0,0) then the coordinates of the red point on the green circle are $(\theta-\sin\theta, 1-\cos\theta)$.



- Here, P is the point on the cycloid and F is the point on the circle.
- The point Q is chosen so that the length DF is the same as the length PQ.
- So if $P = (\theta-\sin\theta, 1-\cos\theta)$ and $F = (\sin\theta, 1-\cos\theta)$ then $Q = (\theta, 1-\cos\theta)$.
- The area of rectangle $OABC$ is 2π. The area to the right of $\{(\theta, 1-\cos\theta) : 0 \leq \theta \leq \pi\}$ is π.
- The area to the left of F and to the right of D, for varying F and D is $\pi/2$.
- So the area to the left of Q and to the right of P, for varying P and Q, is $\pi/2$.

- So the area to the right of P in the rectangle $OABC$ is $3\pi/2$.
- So the total area under the cycloid is 3π.

How would you do it?

How would any of us do it? Using calculus the area under half the cycloid is

$$\int_0^\pi y dx$$

or

$$\int_0^2 (\pi - x) dy.$$

It is possible (just) to write x as a function of y: Since

$$\cos\theta = 1 - y,$$

we have

$$x = \cos^{-1}(1-y) - \sqrt{1-(1-y)^2},$$

which can be integrated, but only using a change of variable — something like $1 - y = \cos\theta$. So we might as well change variable straight away, and consider

$$\int_0^\pi y \frac{dx}{d\theta} d\theta$$

or

$$\int_0^\pi (\pi - x) \frac{dy}{d\theta} d\theta.$$

We have

$$\begin{aligned}\int_0^\pi y \frac{dx}{d\theta} d\theta &= \int_0^\pi (1-\cos\theta)^2 \\ &= \int_0^\pi (1 - 2\cos\theta + \cos^2\theta) d\theta \\ &= \int_0^\pi \left(\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta \\ &= \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^\pi = \frac{3\pi}{2}.\end{aligned}$$

References

- Kline, M. *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, 1972.
- http://www-history.mcs.st-andrews.ac.uk/history/
- http://mathworld.wolfram.com/Cycloid.html