

of the curve under BL, the sum of all the l 's but this is also mentioned incidentally.

To resume, $\frac{l}{a} = \frac{p}{\text{omn. } l} \stackrel{\text{ERROR}}{=} y$, therefore $p = \frac{\overline{\text{omn. } l}}{a} l$. Hence, $\text{omn. } y \frac{l}{a}$ does not mean the same thing as $\text{omn. } y$ into $\text{omn. } l$, nor yet y into $\text{omn. } l$; for, since $p = \frac{y}{a} l$ or $\frac{\overline{\text{omn. } l}}{a} l$, it means the same thing as $\text{omn. } l$ multiplied by that one l that corresponds with a certain p ; hence, $\text{omn. } p = \text{omn. } \frac{\overline{\text{omn. } l}}{a} l$. Now I have otherwise proved $\text{omn. } p = \frac{y^2}{2}$, i. e., $= \frac{\overline{\text{omn. } l^2}}{2}$; therefore we have a theorem that to me seems admirable, and one that will be of great service to this new calculus, namely,

$$\frac{\overline{\text{omn. } l^2}}{2} = \text{omn. } \frac{\overline{\text{omn. } l}}{a} l, \text{ whatever } l \text{ may be;}$$

that is, if all the l 's are multiplied by their last, and so on as often as it can be done, the sum of all these products will be equal to half the sum of the squares, of which the sides are the sum of the l 's or all the l 's. This is a very fine theorem, and one that is not at all obvious.

Another theorem of the same kind is:

$$\text{omn. } xl = x \text{omn. } l - \text{omn. } \text{omn. } l,$$

where l is taken to be a term of a progression, and x is the number which expresses the position or order of the l corresponding to it; or x is the ordinal number and l is the ordered thing.

N. B. In these calculations a law governing things of the same kind can be noted; for, if omn. is prefixed to a number or ratio, or to something indefinitely small, then a line is produced, also if to a line, then a surface, or if to a surface, then a solid; and so on to infinity for higher dimensions.

It will be useful to write \int for omn. , so that

$$\int l = \text{omn. } l, \text{ or the sum of the } l\text{'s.}$$

$$\text{Thus, } \frac{\int l^2}{2} = \int \int \frac{l}{a}, \text{ and } \int xl = x \int l - \int \int l.$$

From this it will appear that a law of things of the same kind

should always be noted, as it is useful in obviating errors of calculation.

N. B. If $\int l$ is given analytically, then l is also given; therefore if $\int \int l$ is given, so also is l ; but if l is given, $\int l$ is not given as well. In all cases $\int x = x^2/2$.

N. B. All these theorems are true for series in which the differences of the terms bear to the terms themselves a ratio that is less than any assignable quantity.

$$\int x^2 = \frac{x^3}{3}$$

Now note that if the terms are affected, the sum is also affected in the same way, such being a general rule; for example,

$\int \frac{a}{b} l = \frac{a}{b} \times \int l$, that is to say, if $\frac{a}{b}$ is a constant term, it is to be multiplied by the maximum ordinal; but if it is not a constant term, then it is impossible to deal with it, unless it can be reduced to terms in l , or whenever it can be reduced to a common quantity, such as an ordinal.

N. B. As often as in the tetragonistic equation, only one letter, say l , varies, it can be considered to be a constant term, and $\int l$ will equal x . Also on this fundamental there depends the theorem:

$$\int \frac{l^2}{2} = \int \int l, \text{ that is, } \frac{x^2}{2} = \int x.$$

Hence, in the same way we can immediately solve innumerable things like this; thus, we require to know what e is, where

$$\int \frac{c}{a} \int l + ba^2 + \int l^3 + \int l^3 = ea^3;$$

we have

$$a^3e = \frac{cx^3}{3} + ba^2x + \frac{x^4}{4} + xa^3.$$

For indeed $\int l^3 = x$, because l is supposed to be equal¹⁹ to a for the purpose of the calculation; $\int \frac{l}{a} = x$.

¹⁸ There is evidently a slip here; l should be x .

¹⁹ This is an instance of the care which Leibniz takes; in the work above l has been the difference for y , and a the difference for x ; he is now integrating an algebraical expression, and not considering a figure at all; hence $l = a$, and a is equal to unity, and therefore $\int l^3 = \int a^3x = a^3x = x$! Thus what is generally considered to be a muddle turns out to be quite correct. The muddle is not with Leibniz, it is with the transcriber. It is certain that these manuscripts want careful republishing from the originals; won't some millionaire pay to have them reproduced photographically in an *edition de luxe*?

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