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Solve the system of linear equations

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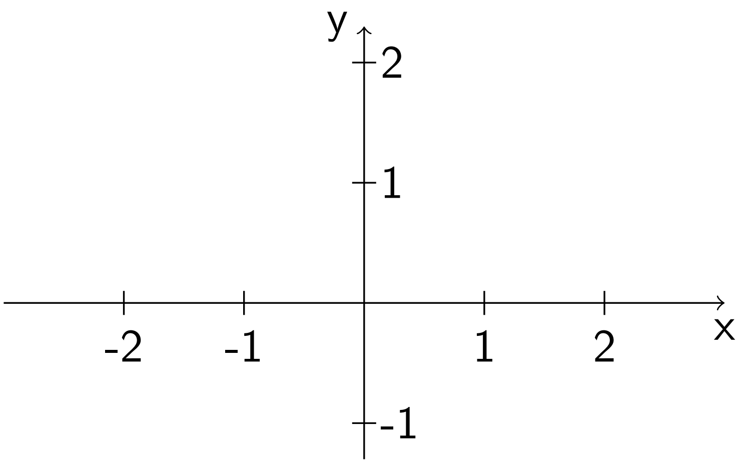

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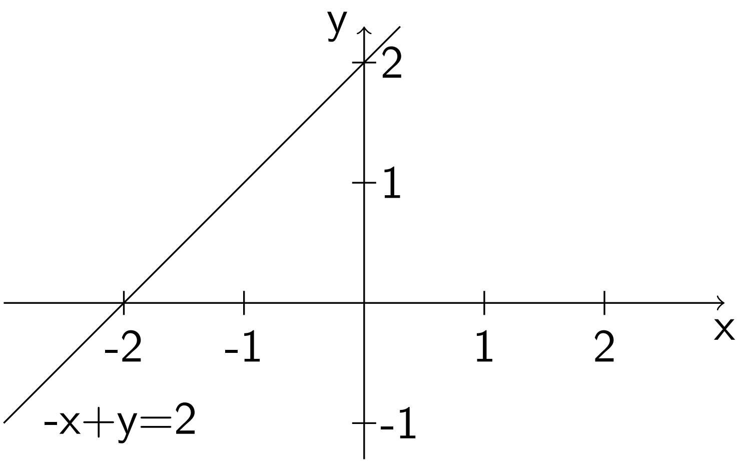

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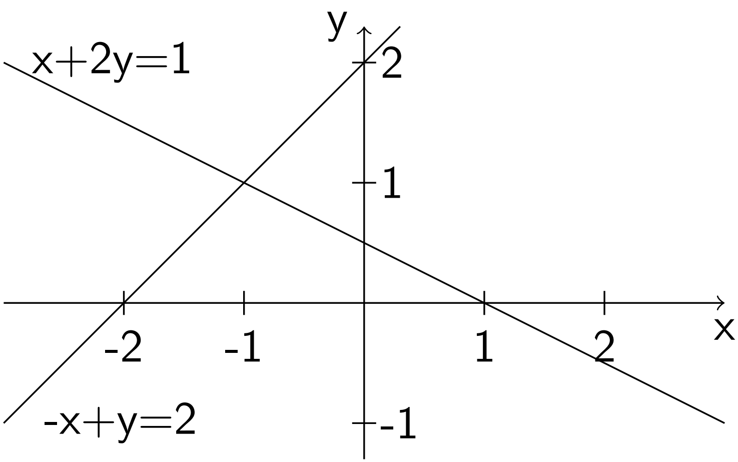

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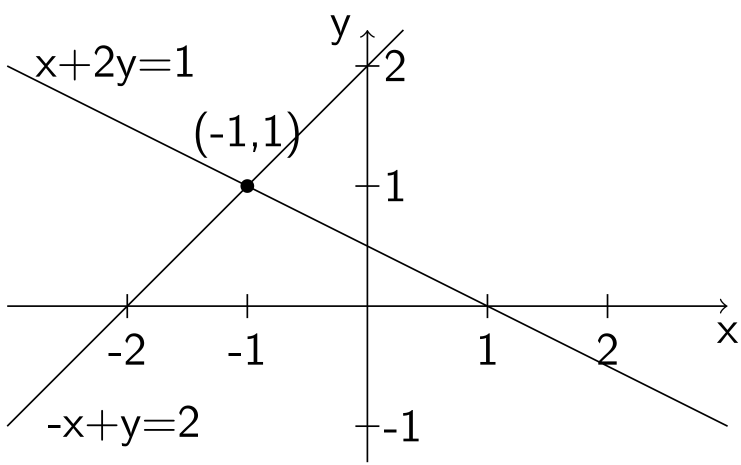

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Linear Algebra develops methods to solve systems of linear equations and tools to analyse such systems of linear equations and their solutions.

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What is Linear Algebra good for?

Linear Algebra provides a theoretical framework in which to attack these problems.

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x_i:	production level in economy sector i
coefficient of x_i in jth row:	part of production that sector j needs from sector i
constant in jth row:	consumers' demand of products of sector i

$$
\begin{aligned}
x_1 &= 0.1588x_1 + 0.0064x_2 + 0.0025x_3 + 0.0304x_4 + 0.0014x_5 + 0.0083x_6 + 0.1594x_7 + 74,000\\
x_2 &= 0.0057x_1 + 0.2645x_2 + 0.0436x_3 + 0.0099x_4 + 0.0083x_5 + 0.0201x_6 + 0.3413x_7 + 56,000\\
x_3 &= 0.0264x_1 + 0.1506x_2 + 0.3557x_3 + 0.0139x_4 + 0.0142x_5 + 0.0070x_6 + 0.0236x_7 + 10,500\\
x_4 &= 0.3299x_1 + 0.0565x_2 + 0.0495x_3 + 0.3636x_4 + 0.0204x_5 + 0.0483x_6 + 0.0649x_7 + 25,000\\
x_5 &= 0.0089x_1 + 0.0081x_2 + 0.0333x_3 + 0.0295x_4 + 0.3412x_5 + 0.0237x_6 + 0.0020x_7 + 17,500\\
x_6 &= 0.1190x_1 + 0.0901x_2 + 0.0996x_3 + 0.1260x_4 + 0.1722x_5 + 0.2368x_6 + 0.3369x_7 + 196,000\\
x_7 &= 0.0063x_1 + 0.0126x_2 + 0.0196x_3 + 0.0098x_4 + 0.0064x_5 + 0.0132x_6 + 0.0012x_7 + 5,000
\end{aligned}
$$

sector 1:	nonmetal household and personal products
sector 2:	final metal products (cars etc.)
sector 3:	basic metal products and mining
sector 4:	basic nonmetal products and agriculture
sector 5:	energy
sector 6:	services
sector 7:	entertainment and miscellaneous products