MATH191: Problem Sheet 9

Due *Monday 6th December*

1. Let $\mathbf{a} = (1, 0, -2)$, $\mathbf{b} = (-1, 2, -1)$, and $\mathbf{c} = (1, -1, -3)$. Calculate the following quantities: $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{c}$, $2\mathbf{a} - 2\mathbf{b} + 3\mathbf{c}$, $|\mathbf{a}|$, $|\mathbf{b}|$, $|\mathbf{a} - \mathbf{c}|$, $\mathbf{a} \cdot \mathbf{b}$, and $\mathbf{b} \cdot (\mathbf{a} - \mathbf{c})$. Find the angle between $\mathbf{a}$ and $\mathbf{b}$, and the angle between $\mathbf{b}$ and $\mathbf{a} - \mathbf{c}$, giving each of the two answers in radians to three decimal places.

2. Let $z_1 = 1 + 2j$ and $z_2 = 1 - 3j$. Calculate $z_1 + z_2$, $z_1 - z_2$, $z_1 z_2$, and $\frac{z_1}{z_2}$. Write down the complex conjugates $\bar{z_1}$ and $\bar{z_2}$, and calculate $\bar{z_1}\bar{z_2}$ and $\frac{\bar{z_1}}{\bar{z_2}}$ *any way you like.*

3. Determine the modulus and argument of the following complex numbers z, and hence write them in the form $z = re^{j\theta}$. Calculate the arguments to three decimal places if you can't write them down exactly (e.g. as $\pi/4$). Keep the moduli in the form $\sqrt{n}$ if they are not whole numbers.

a) $2 + j$; b) -3; c) $1 + j$; d) $-3j$; e) $-3 - 4j$; f) $4j$; g) $-1 + j$.

(Hint: remember that the argument of $z = x + jy$ can't be calculated simply as $\theta = \tan^{-1}(y/x)$: you have to consider the signs of x and y.)

4. Calculate the values of $(1 + j)^n$ for $n \leq 8$.

5. Find all solutions of the following equations in the form $z = a + bj$ Give a and b exactly, whenever possible.

a) $z^2 = -1 - \sqrt{3}j$; b) $z^3 = 64j$.

I will collect solutions at the lecture on Monday 6th December. Any solutions which are not handed in then, or by 5pm that day in the envelope outside Office 516 in the Maths Building will not be marked. (My office, 515, is reached through 516.)