MATH191: Problem Sheet 4

Due *Monday 25th October*

1. For each of the following functions $f(x)$, evaluate

$$\lim_{x\to+\infty} f(x) \qquad \text{and} \qquad \lim_{x\to-\infty} f(x),$$

whenever it is possible to evaluate them.

a) $f(x) = x^3 - 3x^2 + x - 1$; b) $f(x) = \dfrac{x^3 + x^2}{x^2 - 2}$; c) $f(x) = \dfrac{3x^2 - 2x + 3}{2x^2 - 3}$;

d) $f(x) = \dfrac{5x^2 - 2x + 3}{x^3 + 4x^2 - 1}$; e) $f(x) = \cos x$; f) $f(x) = \dfrac{\cos x}{x}$.

2. Differentiate the following functions:

a) $x^3 + 2x^2 - 3x + 2$; b) $x^4 \sin x$; c) $2\sqrt{x} + \cos x$; d) $\dfrac{1}{\sqrt{1+x}}$;

e) $\dfrac{\sin x}{x^2}$; f) $\cos(x^2 + 1)$; g) $\dfrac{1}{(2+3x)^2}$.

Don't guess! Use the rules of differentiation carefully as I did in the lectures. In parts c) and d), remember that $\sqrt{x} = x^{1/2}$, and $\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2}$.

3. Find the equation of the tangent to the graph $y = f(x)$ at the point (x_0, y_0) in each of the following cases:

a) $f(x) = x^2$, $(x_0, y_0) = (3, 9)$; b) $f(x) = x^3$, $(x_0, y_0) = (-1, -1)$.

c) $f(x) = x^2 \cos x$, $(x_0, y_0) = (0, 0)$; d) $f(x) = \dfrac{\sin x}{x}$, $(x_0, y_0) = (\pi, 0)$.

4. Use the binomial theorem to expand the following:

a) $(1 + x)^4$; b) $(1 + 2x)^4$; c) $(1 - x)^4$.

I will collect solutions at the lecture on Monday 25th October. Any solutions which are not handed in then, or by 5pm that day in the envelope outside Office 516 in the Maths Building will not be marked. (My office, 515, is reached through 516.)