# MATH191: Practice Sheet 8

**1.** Let $f(x)$ be defined by
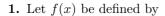
$$f(x) = \begin{cases} x^2 - 4x & \text{if } x \geq 0 \\ \frac{x}{x+1} & \text{if } x < 0, \ x \neq -1. \end{cases}$$

Differentiate $f(x)$ for $x \neq 0, -1$. Find and classify any stationary points, determine any zeros of $f(x)$, and any horizontal and vertical asymptotes. Is $f(x)$ continuous at $x = 0$? Is it differentiable at $x = 0$? Sketch the graph of $f(x)$.

**2.** Differentiate the following functions:

a) $x^2 e^x$;     b) $\dfrac{x^2}{e^x}$;     c) $e^{x^2+1}$;     d) $\cos(x^2 + 1)$;     e) $(3x + 2)^{-1/2}$.

**3.** Evaluate the following integrals, giving your answers exactly if possible, otherwise to 3 decimal places:

a) $\displaystyle\int_0^1 (2x^2 - 4x + 2)\, dx$;     b) $\displaystyle\int_{-2}^{-1} \left( e^{-x} - \frac{1}{x} \right) dx$;     c) $\displaystyle\int_{-\pi/2}^{\pi/2} (\cos(2x) + \sin(x))\, dx$.

**4.** Find the indefinite integral $\int f(x)\, dx$ of each of the following functions $f(x)$.

a) $x^3 + 1$;     b) $e^{2x+1} + \cosh(3x - 2)$;     c) $\dfrac{1}{\sqrt{x}} + \dfrac{1}{x}$;     d) $\sin^2 x$.

(*Hint: For part d), try using the trigonometric identity* $\cos 2x = 1 - 2\sin^2 x$.)