MATH191: Practice Sheet 4

1. For each of the following functions $f(x)$, evaluate

$$\lim_{x\to+\infty} f(x) \qquad \text{and} \qquad \lim_{x\to-\infty} f(x),$$

whenever it is possible to evaluate them. Answers of $+\infty$ and $-\infty$ are allowed in this question.

a) $f(x) = x^2 + 2x - 1$; b) $f(x) = \dfrac{x^3 + 3x}{x^2 + 1}$; c) $f(x) = \dfrac{2x^2 + 3x + 1}{x^2 - 3}$;

d) $f(x) = \dfrac{2x^2 + 3x + 1}{x^3 - 4x^2 + 1}$; e) $f(x) = \sin x$; f) $f(x) = \dfrac{\sin 2x}{x}$.

2. Differentiate the following functions:

a) $2x^3 - 3x^2 + 2$; b) $x^2 \cos x$; c) $\dfrac{3}{\sqrt{x}}$; d) $\sqrt{1+x}$;

e) $\dfrac{2x^2 + 1}{\cos x}$; f) $\sin(3x^2 - 2)$; g) $\dfrac{1}{(2x-1)^3}$.

Hints: **Don't guess.** *Use the rules of differentiation carefully as I did in the lectures. In parts c) and d), remember that* $\frac{1}{\sqrt{x}} = x^{-1/2}$, *and* $\sqrt{1+x} = (1+x)^{1/2}$.

3. Find the equation of the tangent to the graph $y = f(x)$ at the point (x_0, y_0) in each of the following cases:

a) $f(x) = x^2$, $(x_0, y_0) = (-2, 4)$; b) $f(x) = x^3$, $(x_0, y_0) = (1, 1)$.

c) $f(x) = x + \sin x$, $(x_0, y_0) = (0, 0)$; d) $f(x) = x \cos x$, $(x_0, y_0) = (\pi/2, 0)$.

4. Use the binomial theorem to expand the following:

a) $(1+x)^5$; b) $(2+x)^5$; c) $(1-x)^5$.