MATH191: Practice Sheet 3

1. Find the general solution of the equation

$$3\cos\theta + 2\sin\theta = 1.$$

correct to 3 d.p..

2. In a) and b), convert from polar to Cartesian coordinates. In c) and d), convert from Cartesian to polar coordinates. Give the answers exactly in terms of π and square roots of intergers, where possible. Where this is not possible, give answers correct to 4 d.p.

a) $(r,\theta) = (1,\pi/3)$; b) $(r,\theta) = (2,-\pi/4)$; c) $(x,y) = (\sqrt{2},1)$; d) $(x,y) = (-3,4)$.

3. Let

$$f(x) = \frac{\sin(x) - x}{x^3}.$$

Calculate $f(x)$ for each of the values $x = 0.1$, -0.1, $0.01$and -0.01. What does this suggest the value of $\lim_{x\to 0} f(x)$ to be? (If you don't recognize the number you're getting, try hitting the `1/x` (or x^{-1}) button on your calculator.)

4. For each of the following functions $f(x)$, evaluate the limit

$$\lim_{x\to a} f(x)$$

for the given value of a, or explain why the limit doesn't exist.

a) $f(x) = x^3 + 3,\ a = 1$; b) $f(x) = \dfrac{x^2-4}{x-2},\ a = 2$; c) $f(x) = \dfrac{x^2-5}{x-2},\ a = 2$;

d) $\lim\limits_{x\to\pm\infty} f(x) = \dfrac{x^2-5}{(x-2)^2}$; e) $f(x) = \dfrac{\sin 3x}{x},\ a = 0$; f)$f(x) = \dfrac{x}{|x|}.,\ a = 0$

(In part e), you may use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$.)