Math191 Class Test Monday 8 November 2010

- The exam will be **45 minutes** long.

- No books or notes are allowed.

- Attempt as many questions as you can. **FULL MARKS** will be given for complete answers to **ALL** seven questions. The marks available for each question are indicated in square brackets in the right margin.

- Please turn your **MOBILE PHONES OFF** and **DON'T TALK**.

1. State the domain and range of the following functions:

a) $f(x) = \cos(2x) - 1$

b) $f(x) = |x - 2|$

[6 marks]

2. Let
$$f(x) = \frac{3 - x}{1 - x}.$$
Find the inverse function $f^{-1}(x)$. State the domain and range of f (NOT the inverse function) and sketch its graph, marking any horizontal or vertical asymptotes, and any zeros.

[10 marks]

3.

a) Find the exact value of $\sin^{-1}(-\frac{\sqrt{3}}{2})$.

b) Give the general solution of the equation $\sin x = -\frac{\sqrt{3}}{2}$.

[6 marks]

4. In this question, full marks will only be awarded for *exact* answers (in terms of $\pi, \sqrt{2}$ etc.) and not for approximations to any number of decimal places.

a) Convert $(\sqrt{2}, 5\pi/4)$ from polar to Cartesian coordinates.

b) Convert $(-1, \sqrt{3})$ from Cartesian to polar coordinates.

[5 marks]

5. Determine whether the following limits exist. Where they exist, evaluate them.

a) $\displaystyle\lim_{x \to \pm\infty} \frac{x^2 - 2x + 2}{x^3 - 2}$

b) $\displaystyle\lim_{x \to -3} \frac{x^2 + 4x + 3}{x^2 + 2x - 3}$

[6 marks]

6.

Differentiate the following functions. In part a), also find the tangent line through the point $(1, 1)$.

a) $f(x) = x^3 - 2x + 2$

b) $f(x) = \cos(4x)$

c) $f(x) = \dfrac{\cos x}{x^2 + 1}$

[11 marks]

7.

a) Find the Maclaurin series of $f(x) = (1 + x)^{-2}$

b) Hence, or otherwise, find the Maclaurin series of $(1 + x^2)^{-2}$

[6 marks]