MATH105 Numbers and Sets

The concepts of definitions, theorems (hypothesis and conclusion, converse and contrapositive), proofs (direct proof and proof by contradiction), examples and counter examples, quantifiers and negation will be introduced gradually via examples throughout the module.

[3 lectures] Basic propositional logic (exemplified by 'real world' logic puzzles).

[10 lectures] Natural numbers, Peano axioms. Integers. Principle of mathematical induction and proof by induction. Greatest common divisors. Euclid's algorithm. Fundamental Theorem of Arithmetic (unique factorisation into primes), and applications: greatest common divisors and least common multiples, numbers of divisors. Primes. Sieve of Eratosthenes. Existence of infinitely many primes.

[5 lectures] Sets and maps. Injectivity, surjectivity and bijectivity. Intersection and union, conditional and constructive definitions of sets, principle of inclusion-exclusion.

[2 lectures] Equivalence relations and quotients. Simple examples from 'real life' and arithmetic.

[9 lectures] Definition of rational numbers via integer numbers. Construction of real numbers via Dedekind cuts. Irrational numbers; proof of existence, examples. Sequences. Completeness of the real numbers as their key property. (Connect with Math101.)

[3 lectures] Countability. Cantor's diagonal argument. Existence of transcendental numbers.

[2 lectures] Construction of complex numbers from the reals. Basic properties (connect with Math103). Statement and sketch proof of Fundamental Theorem of Algebra.

[2 lectures] Revision

Recommended Texts

P.J.Eccles, An introduction to mathematical reasoning. Cambridge University Press, 1997. £25 in Blackwells (discounted), From £12 on Amazon for used copies, 10 copies in library.

J.F. Humphreys and M.Y. Prest, Numbers, groups and codes. Cambridge University Press, 1989 and 2004. £37 in Blackwells (discounted), also used in MATH142, from £13 on Amazon for used copies, electronic networked copy in library, plus 10 hard copies.

For background: V. Bryant. Yet another introduction to analysis. Cambridge University Press, 1990, £34 in Blackwells, from £19 on Amazon for used copies, 10 copies in library.