Starter, Lecture 1

What is wrong with this logic?

MATH105: Lecturer and lectures

- Lecturer: Prof. S. M. Rees
- Room 515, Maths & Oceanography Building,

  reached through 516.

  Lectures:

  - Wednesday 11-11:50 p.m. in CTH-B (Lecture Theatre B, Central Teaching Hub),
  - Thursday 9-9:50 a.m., in CHEM-GOS(Gossage Lecture Theatre, Chemistry),
  - Friday 10-10:50 a.m. in CHEM-GOS (Gossage Lecture Theatre, Chemistry),

Problem classes

- Three groups, one with Prof. Mary Rees, one with Dr. Joel Haddley and one with Prof Viacheslav (Slava) Nikulin.
- All groups 3-3:50 p.m. on Thursdays, starting on 2nd October.
- Prof Rees's group in REN-LT1 (Lecture Theatre 1, Renshaw Building),

- Dr Haddley's group in MATH029 (Room 029, Ground Floor, Mathematics Building),
- Prof. Nikulin's group in CTH-LTD (Lecture Theatre D, Central Teaching Hub).

Office Hours

- Wednesday 12:00-1:00
- Friday 2:00-4:00
- both in room 515, Mathematics Building.

Set Work

- Problems circulated on Wednesday.
- Solutions collected following Wednesday.
- Each set of problems contributes about 1% to final mark.
- No marks after 5pm Wednesday.
- Illness: both the Certificate of Illness and Mitigating Circumstances forms can be downloaded from the department's Current Students' webpage

  http://www.liv.ac.uk/maths-current-students/

  and all forms be submitted to the MathStudentSupport office, room G33, Maths Building.

Assessment

- 90% from two-and-a-half hour exam in January.
- 10% from weekly problem sheets.

Only the university-approved calculator can be used in the exam.

Recommended Books

- P.J.Eccles, An introduction to mathematical reasoning. Cambridge University Press, 1997. £25 in Blackwells (discounted), From £12 on Amazon for used copies 10 copies in library.
- J.F. Humphreys and M.Y. Prest, Numbers, groups and codes. Cambridge University Press, 2004/1989. £37 in Blackwells (discounted), also used in MATH142, from £13 on Amazon for used copies, electronic networked copy in library.
- For background: V. Bryant. Yet another introduction to analysis. Cambridge University Press, 1990, £34 in Blackwells, from £19 for used copies on Amazon, 10 copies in library.

Help

- Thursday problem class
- End of Lectures
- Office Hours (or other times by arrangement).
- Recommended books

The lectures are for you. Make good use of them:

- ask questions
- tell me if I'm too fast / slow etc
- and please **DON'T TALK** in lectures - except to me.

Wrong Mathematics and Non-mathematics

If someone writes down:

$$y = x^2 \sin x.$$

So

$$\frac{dy}{dx} = 2x \cos x$$

then that is wrong. (Can you see what they did?).

But if they write

$$x^2 \sin x$$

$$2x \sin x + x^2 \cos x$$

$$0, \quad 2 \sin x + x \cos x$$

then that is cryptic (to put it charitably).

- In the first case two *statements* were written down (of which the second is wrong if it is the same y).
- In the second case a list of *expressions* was written down. Since they are written down consecutively we may be able to fill in the chain of thought being expressed. But otherwise, as expressions, they do not make sense.
- Mathematics should be understandable by someone other than the person who wrote it.
- We should also try to avoid ambiguity in what we write.

Back to: What

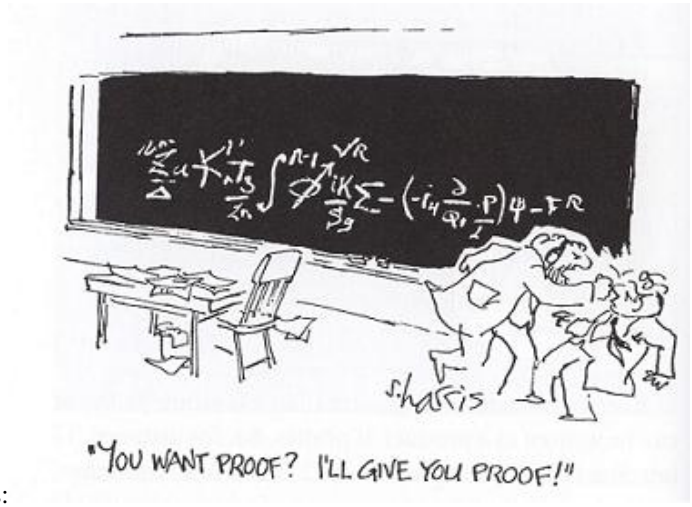
is wrong with this?

It is the difference between the statement

- If C is a cat then C has four legs (true)
  and its *converse*
- If C has four legs then C is a cat (false).

Similarly for real numbers

- If $x > 1$ then $x^2 > 1$ (true) and
- If $x^2 > 1$ then $x > 1$ (false).



We shall also discuss *proofs* of theorems:

Syllabus of MATH105

- Basic propositional logic
- Natural numbers
- Sets and maps
- Equivalence relations and quotients
- Rational numbers and real numbers
- Countability
- Complex numbers

Here is a list of *statements* which may be true or false.

1. $3 \leq 5$.
2. $2 \leq 2$.
3. $2 + 2 = 3$.
4. If $x < -2$ then $x^2 > 4$.
5. If $x^2 > 4$ then $x < -2$.
6. $0 < \frac{1}{2}$ and $2 < 1$.
7. $0 < \frac{1}{2}$ or $2 < 1$.
8. There is no integer n with $n^2 = 3$.

9. There is no rational number x with $x^2 = 3$

The following are *not* statements.

- $2 + 3$
- 2^{32}
- $x^2 + 1$

Free variables

- Each of the statements that was written was either true or false.
- But there are other statements for which one cannot say without more information whether they are true or false
- An exact analogy in English is difficult but consider

  "Elephants roam the forests"

  (Which forests? Which elephants? Where? When?)