MATH105 Problem Sheet 1: Introductory

Due Wednesday 8th October

1. There is a saying that

St Swithins Day, if it does rain
Full forty days, it will remain
St Swithins Day, if it be fair
For forty days, twill rain no more.

St Swithin's Day is 15 July. Assume that this saying is true. In each of the following, give the correct answer "Yes" or "No" or "Impossible to say"

(a) If it rains on 15 July, will it rain on 20 July?

(b) If it rains on 15 July, will it rain on 30 August?

Now assume that
St Swithins Day, if it does rain
Full forty days, it will remain
and *nothing more.* Answer the questions in the same way.

c) If it remains dry on 15 July, will it rain on 9 August?

d) If it rains on 9 August, was it raining on 15 July?

e) If it is dry on 9 August, was it raining on 15 July?

2.

Write each of the following statements in ordinary English and determine whether each one is true.

(a) $1 < 2 \wedge 2 < 3$.

(b) $1 < 2 \vee 2 < 3$.

(c) $1 < 2 \vee 2 < 1$.

(d) If x and y are real numbers, $x < y \Rightarrow 2x < 2y$.

(e) If x, y and z are real numbers, $xz < yz \Rightarrow x < y$.

(f) $x^2 + 3x - 4 = 0 \Leftrightarrow x = 1 \wedge x = -4$.

(g) If x is a real number $x^2 \leq 0 \Rightarrow x = 0$.

3. Negate each of the following, using logical symbols where possible.

a) $x^2 + 2x - 1 > 0$.

b) $-1 < x < 2$.

c) $x + y > 0 \wedge x^2 + y^2 < 1$.

d) $x + y > 0 \vee x^2 + y^2 < 1$.

4. (a) Consider the following 'Theorem' and 'Proof'. Remember that $x \leq y$ means that either $x < y$ or $x = y$. Thus for example $2 \leq 2, 2 \leq 3$ are both true, while $3 \leq 2$ is false.

'Theorem' Let a and b be real numbers. If $(a+b)^2 \leq 4ab$ then $a = b$.

'Proof' Suppose that $a = b$. Then $(a+b)^2 = 4a^2$ and $4ab = 4a^2$, and since $4a^2 \leq 4a^2$ is true the theorem is true. $\square$

(i) Is the 'Proof' correct? Give clear reasons for your answer.
(ii) Is the 'Theorem' true? Again give clear reasons.

(b) Answer the same two questions for the following:

'Theorem' Let a and b be real numbers. If $(a+b)^2 \leq 6ab$ then $a = b$.

'Proof' Suppose that $a = b$. Then $(a+b)^2 = 4a^2$ and $6ab = 6a^2$, and since $4a^2 \leq 6a^2$ is true the theorem is true. $\square$

I will collect solutions at the lecture on Wednesday 8th October. Any solutions which are not handed in then, or by 5pm that day in the folder outside room 516 will not be marked.

Possible tutorial problems

Some or all of these may be used in the problem class, if desired. They can also used as practice problems, either for practising for the homework problems, or for later practice during revision, or both.

5. Write each of the following statements in ordinary English, and determine whether each of them is true.

a) $2 < 3 \wedge 3 < 2$.

b) $2 < 3 \vee 3 < 2$.

c) $2 < 3 \wedge 4 > 3$.

d) If x and y are real numbers, $x < y \;\Rightarrow\; -y < -x$.

e) If x and y are real numbers, $x < y \Rightarrow x^2 < y^2$.

6. Negate each of the following, using logical symbols where possible.

a) $3x + 1 \geq 0$.

b) $1 \leq x < 3$.

c) $x - y < 0 \wedge xy > 1$.

d) $x - y < 0 \vee xy > 1$.

7. (a) Consider the following 'Theorem' and 'Proof'.

'Theorem' Let a be a real number. If $a^3 > 2a^2$ then $a > 2$.

'Proof' Suppose that $a > 2$. Then $a^2 \times a > a^2 \times 2$. Hence $a^3 > 2a^2$ and the theorem is true. $\square$

(i) Is the 'Proof' correct? Give clear reasons for your answer.
(ii) Is the 'Theorem' true? Again give clear reasons.

(b) Answer the same two questions for the following:

'Theorem' Let a be a real number. If $a^3 > 4a$ then $a > 2$.

'Proof' Suppose that $a > 2$. Then $a^2 > 2 \times 2 = 4$ and so $a^3 = a \times a^2 > a \times 4 = 4a$. So the theorem is true. $\square$