Practice Class Test Solutions MATH102 2008

1. The Taylor series is

$$\sum_{k=0}^{\infty} \frac{(-1)^k 2^{2k+1} x^{2k+1}}{(2k+1)!} = 2x - \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 \cdots$$

No working is required. [3 marks]

2.

$$f'(x) = \frac{1}{2}x^{-1/2}, \quad f''(x) = \frac{-1}{4}x^{-3/2}, \quad f^{(3)}(x) = \frac{3}{8}x^{-5/2},$$

$$f(4) = 2, \quad f'(4) = \frac{1}{4}, \quad f''(4) = \frac{-1}{32}.$$

So

$$P_2(x) = 2 + \frac{1}{4}(x-4) + \frac{-1}{64}(x-4)^2.$$

[4 marks]

$$P_2(3) = 2 - \frac{1}{4} - \frac{1}{64} = 1.734375.$$

[1 mark]

$$R_2(3) = \frac{3}{8 \times 3!}c^{-5/2}(-1)^3$$

for some $c \in [3,4]$.

2 marks

So

$$|R_2(3)| \leq \frac{1}{16 \times 3^{5/2}} = 0.004009377.. < .005.$$

So

$$|\sqrt{3} - 1.734375| \leq 0.005.$$

This is true because $\sqrt{3} = 1.732050808...$

[2 marks]

3.

$$\int \frac{-dy}{y^2} = \int \frac{dx}{x}.$$

So

$$\frac{1}{y} = \ln x + C$$

[3 marks]

Putting $x = 1$, $\frac{1}{2} = C$. So

$$y = \frac{2}{2\ln x + 1}.$$

[2 marks]

4.

$$\frac{dy}{dx} + \frac{-1}{x}y = x.$$

$$\int \frac{-dx}{x} = -\ln x = \ln(x^{-1}).$$

So the integrating factor is $\exp(\ln x^{-1}) = x^{-1}$.

[3 marks]

So the equation becomes

$$\frac{d}{dx}(x^{-1}y) = 1.$$

So

$$x^{-1}y = x + C$$
$$y = x^2 + Cx.$$

[2 marks]

Putting $x = 2$, $1 = 4 + 2C$ and $C = -\frac{3}{2}$. So

$$y = x^2 - \frac{3}{2}x.$$

[2 marks]

5. Trying $y = e^{rx}$ we obtain

$$r^2 - 4r + 5 = 0$$

So $r = 2 \pm i$. So the general solution is

$$y = Ae^{(2+i)x} + Be^{(2-i)x} = e^{2x}(C\cos x + D\sin x).$$

[4 marks]

6. Trying $y = e^{rx}$ for a solution to the equation with e^{2x} on the righthand side replaced by 0, we obtain

$$r^2 + 2r + 1 = 0$$

So $r = -1$. So the complementary solution is $y_c = Ae^{-x} + Bxe^{-x}$.

[3 marks]

Try $y = Ce^{2x}$ for a particular solution. Then $dy/dx = 2e^{2x}$ and $d^2y/dx^2 = 4e^{2x}$. So

$$(4 + 4 + 1)e^{2x} = Ce^{2x}$$

and $C = \frac{1}{9}$. So the general solution is

$$y = Ae^{-x} + Bxe^{-x} + \frac{1}{9}e^{2x}.$$

[3 marks]

7. Take limits along the axes, for example.

$$\lim_{x\to 0,\ y=0} \frac{x^2 - xy}{x^2 + y^2} = \lim_{x\to 0} \frac{x^2}{x^2} = 1,$$

$$\lim_{y\to 0,\ x=0} \frac{x^2 - xy^2}{x^2 + y^2} = \lim_{y\to 0} \frac{0}{y^2} = 0.$$

So the limits along the axes are different and the limit as $(x, y) \to (0, 0)$ does not exist.

[4 marks]