- For each of the following equations, decide which of the three methods we have learnt can be used to solve them

  (in principle).

- It might be possible to use more than one method,

  or none.

- The three methods are:

sep	Separation of variables
integ fac	Integrating factor
$y = vx$	$y = vx$ method.

1.
$$xy + x\frac{dy}{dx} = y,$$

2.
$$(x^2y - 3xy^2)\frac{dy}{dx} = y^3 + x^3,$$

3.
$$e^{y/x} + \sin(x/y)y' = \frac{x+y}{x-y},$$

4.
$$e^x y + y' \sin x = x^3 \tan x,$$

5.
$$2xyy' = x^2 - y^2,$$

6.
$$x^2yy' + 3xy^2 + 2yy' = 5.$$

For
$$xy + x\frac{dy}{dx} = y,$$
we can use two methods,
separation of variables
and the integrating factor method.
Can we solve it in practice?
For
$$(x^2y - 3xy^2)\frac{dy}{dx} = y^3 + x^3,$$
we can use the $y = vx$ method.
Can we solve it in practice?
For
$$e^{y/x} + \sin(x/y)y' = \frac{x+y}{x-y},$$

we can use $y = vx$.

Can we solve it in practice? For

$$e^x y + y' \sin x = x^3 \tan x,$$

we can use the integrating factor method.

Can we solve it in practice? For

$$2xyy' = x^2 - y^2,$$

we can use $y = vx$.

Can we solve it in practice? For

$$x^2 yy' + 3xy^2 + 2yy' = 5.$$

none of the three methods works,

but there is a simple trick to reduce to integrating factor method.

Can you spot it?

This equation has been changed from the one in lectures, which was, in fact, separable, as was pointed out by someone at the end of the lecture.

Overall the answers are:

1. sep and integ fac,
2. $y = vx$,
3. $y = vx$,
4. sep,
5. $y = vx$,
6. none of the three directly, but after a change of variable $z = y^2$, sep can be used

Put $z = y^2$. Then $2yy' = z'$. So

$$x^2 yy' + 3xy^2 + 2yy' = 5.$$

becomes

$$(x^2 + 2)z' + 6xz = 10.$$

The standard form is

$$z' + \frac{6x}{x^2 + 2} = \frac{10}{x^2 + 2}.$$

Then the integrating factor is $(x^2 + 2)^3$, and the equation becomes

$$\frac{d}{dx}(z(x^2 + 2)^2) = 10(x^4 + 4x^2 + 4),$$

which can be solved.