- For each of the following equations, decide which of the three methods we have learnt can be used to solve them

  (in principle).

- It might be possible to use more than one method,

  or none.

- The three methods are:

sep	Separation of variables
integ fac	Integrating factor
$y = vx$	$y = vx$ method.

1.
$$xy + x\frac{dy}{dx} = y,$$

2.
$$(x^2y - 3xy^2)\frac{dy}{dx} = y^3 + x^3,$$

3.
$$e^{y/x} + \sin(x/y)y' = \frac{x+y}{x-y},$$

4.
$$e^x y + y' \sin x = x^3 \tan x,$$

5.
$$2xyy' = x^2 - y^2,$$

6.
$$x^2yy' + 3xy^2 + 2yy' = 5.$$

For
$$xy + x\frac{dy}{dx} = y,$$
we can use two methods,
separation of variables
and the integrating factor method.
Can we solve it in practice?
For
$$(x^2y - 3xy^2)\frac{dy}{dx} = y^3 + x^3,$$
we can use the $y = vx$ method.
Can we solve it in practice?
For
$$e^{y/x} + \sin(x/y)y' = \frac{x+y}{x-y},$$

we can use $y = vx$.

Can we solve it in practice? For

$$e^x y + y' \sin x = x^3 \tan x,$$

we can use the integrating factor method.

Can we solve it in practice? For

$$2xyy' = x^2 - y^2,$$

we can use $y = vx$.

Can we solve it in practice? For

$$x^2yy' + 3xy^2 + 2yy' = 5.$$

none of the three methods works,

but there is a simple trick to reduce to integrating factor method.

Can you spot it?

Overall the answers are:

1. sep and integ fac,
2. $y = vx$,
3. $y = vx$,
4. sep,
5. $y = vx$,
6. none of the three directly, but after a change of variable $z = y^2$, sep can be used

Put $z = y^2$. Then $2yy' = z'$. So

$$x^2yy' + 3xy^2 + 2yy' = 5$$

becomes

$$(x^2 + 2)z' + 6xz = 10.$$

The standard form is

$$z' + \frac{6xz}{x^2+2} = \frac{10}{x^2+2}.$$

Then the integrating factor is $(x^2 + 2)^3$, and the equation becomes

$$\frac{d}{dx}(z(x^2+2)^2) = 10(x^4 + 4x^2 + 4),$$

which can be solved.