MATH 102 Key skills Exercise

- This is addressed at the key skills of communication and improving own learning.

- It contributes 5% towards the final module mark.

- Submit your answers to your Tutor by **Monday** 14 April

The other side of this sheet contains a selected list of past paper questions and the number of marks assigned to each question. The other sheet contains photocopies of examination attempts at the questions.

Your tasks are as follows:

1. Create model solutions with a marking scheme.

2. Mark the photocopied examination scripts, clearly indicating your procedures.

3. Write brief comments addressed to the candidates on their errors. Assume that you are **teaching at a distance** and your written explanations are your only contact with the candidate.

2. Find the derivative of the function

$$\frac{(2x^2+5x-5)}{(x^2+7)}.$$

[3 marks]

3. Use integration by parts to evaluate the integral

$$\int_0^{\frac{\pi}{2}} x\cos x\ dx.$$

[6 marks]

Further use the substitution $u = x^2$ to evaluate

$$\int_0^{\frac{\pi}{2}} x\cos x^2\ dx.$$

[3 marks]

10. (a) The periodic function $f(\theta)$ is defined by

$$f(\theta) = \begin{cases} -\theta - \pi & -\pi < \theta < 0 \\ \pi - \theta & 0 < \theta < \pi \end{cases} \text{ and } f(\theta + 2\pi) = f(\theta).$$

State whether $f(\theta)$ is an even or odd function or neither. Sketch $f(\theta)$ for $-4\pi < \theta < 4\pi$. [4 marks]

12. (a) Find the scalar triple product $\mathbf{a}.(\mathbf{b} \times \mathbf{c})$ for the vectors:

$$\mathbf{a} = 3\mathbf{i} - 6\mathbf{j} + \mathbf{k},\quad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k},\quad \mathbf{c} = \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}.$$

[7 marks]

7. $\frac{2x^2+5x-5}{x^2+7} = \frac{u}{v}$ $\quad \frac{du}{dx} = 4x+5$ $\quad \frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$$

$$= \frac{(2x^2+5x-5)\cdot 2x - (x^2+7)\cdot(4x+5)}{(x^2+7)^2}$$

$$= \frac{(4x^3+10x^2-10x) - (4x^3+28x+5x^2+35)}{x^4+49}$$

$$= \frac{5x^2-38x+35}{x^2+49}$$

this margin

3) $\int_0^{\pi/2} x\cos x \, dx$ $\quad u = x$ $\quad v = -\sin x$

$\frac{du}{dx} = 1$ $\quad \frac{dv}{dx} = \cos x$

$uv - \int v\frac{du}{dx}$

$\Rightarrow -x\sin x - \int -\sin x \cdot 1 \, dx$

$\Rightarrow -x\sin x - \cos x$

$\therefore \left[-x\sin x - \cos x\right]_0^{\pi/2}$

$\Rightarrow (-\pi/2 \sin \pi/2 - \cos \pi/2) - (-0\sin 0 - \cos 0)$

$\Rightarrow 1/2 - 1$

$\Rightarrow -1/2$.

Using sub $u = x^2$

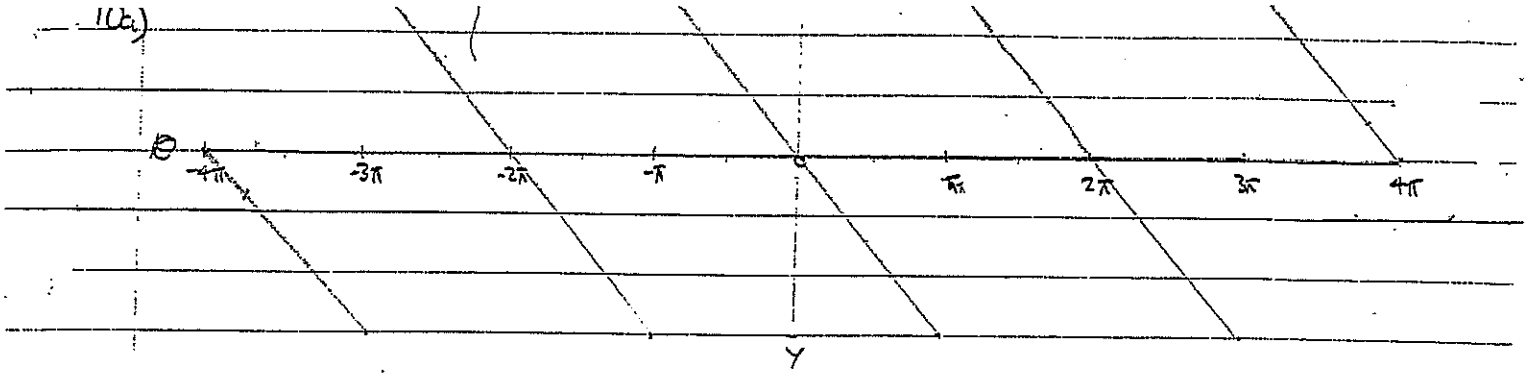
Solution becomes $\left[-x\sin u - \cos u\right]_0^{\pi/2}$

$\Rightarrow (-\pi/2 \sin(\pi/2)^2 - \cos(\pi/2)^2) - (-0\sin 0^2 - \cos 0^2)$.

$= (-0.98 - -0.78) - 1 = -0.2 - 1 = -1.2$

$= -0.2 - 1 = -1.2$

10(b)



This is an odd function since it is not reflected in the Y-axis.

12) a) $a \cdot (b \times c)$

$b \times c = 3i - 2j - 3k \times i + 4j + 2k$

$b \times c = 4i + 2j - k$

$a \cdot (b \times c) = 3i - 6j + k \cdot 4i + 2j - k$

$\theta = \cos^{-1}(ll' + MM' + NN')$

$\theta = \cos^{-1}[(0.493 \times 0.57) + (0.13 \times 0.436) + (0.147 \times -0.214)$

$\theta = \cos^{-1}(0.38 + (-0.0567) - 0.032)$

$\theta = \cos^{-1} 0.29$

$\theta = 73.14°$

$a(b \times c) = 3i - 6j + k \cdot 4i + 2j - k \cos 73.14$

$= 7i - 4j \cos 73.14$

$= (7^2 + 4^2)^{1/2} \cos\theta$

$= 8.06 \cos\theta$

$a \cdot (b \times c) = 2.086$