

Formula for the Centroid of R

The area is

$$A = \int \int_R dx dy = \int \int_R dy dx.$$

The centroid is the centre of mass if mass on R is uniformly distributed. The formula for the centroid is

$$(\bar{x}, \bar{y}),$$

where

$$\bar{x} = \frac{1}{A} \int \int_R x dx dy = \frac{1}{A} \int \int_R x dy dx,$$

and

$$\bar{y} = \frac{1}{A} \int \int_R y dx dy = \frac{1}{A} \int \int_R y dy dx.$$

Formula for the centre of mass with density $\rho(x, y)$.

If the mass has density $\rho(x, y)$, the *weight* of R is

$$W = \int \int_R \rho(x, y) dx dy = \int \int_R \rho(x, y) dy dx.$$

The formula for the centre of mass is

$$(\bar{x}, \bar{y}),$$

where

$$\bar{x} = \frac{1}{W} \int \int_R x \rho(x, y) dx dy = \frac{1}{W} \int \int_R x \rho(x, y) dy dx,$$

and

$$\bar{y} = \frac{1}{W} \int \int_R y \rho(x, y) dx dy = \frac{1}{W} \int \int_R y \rho(x, y) dy dx.$$