Mathematics Foundation Module III: MATH102
Tutorial Problems 2008

Each week, problems will be set from this list. You should study these problems *before* your tutorial. The standard tutorial times are on Thursdays, but some variation is possible. Any difficulties with the problems will be discussed during the tutorial, and final versions of your solutions are to be delivered to your tutor on the following Monday. They will then be marked and returned to you at the next tutorial. Solutions will be handed out on the Thursday 3 p.m. lecture after the Monday hand-in.

There will be one class test during the semester, during Week 6. No tutorial problems are to be handed in during that week, but "revision problems" will be suggested which should help you prepare for the test. Work on problems during the course is the key to success in the final examination in May. Not all the questions will be set for homework. The remainder are suitable for extra practice, and may be used in tutorials.

If you lose this problem collection, another copy is available from VITAL or directly from http://www.liv.ac.uk/∼maryrees/math102.html. For later reference, a number of recent MATH102 exams are available from VITAL, on this webpage and from the www.liv.ac.uk/maths webpages.

A brief syllabus of the module, and details of the recommended text, the same as for MATH101, can be found at the end of this problem collection.

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1. Find the first four derivatives of $f(x) = \ln x$ and write down the expression for the n'th derivative $f^{(n)}(x)$. Obtain the infinite Taylor series for $f(x)$ at $x = 1$.

2. Find the Taylor polynomials of orders 0,1,2 at x=a generated by the functions $f(x)$ in the cases:
(i) $f(x) = \frac{1}{x}$ at $a = 2$, (ii) $f(x) = \sqrt{x+4}$ at $a = 0$,
(iii) $f(x) = \sin 2x$ at $a = 0$, (iv) $f(x) = \cos x$ at $x = \pi$,
(v) $f(x) = \sin 2x$ at $a = \pi/4$, (vi) $f(x) = \sin 2x$ at $a = \pi/2$.

3. Find the infinite Taylor series of $f(x) = \sin x$ at $x = \pi/2$ and of $g(x) = \cos x$ at $x = 0$. Show also that $\sin(x+\pi/2) = \cos x$. How do you explain the similarlity of the two Taylor series?

4. Obtain near $x = 0$ the degree three approximation $P_3(x)$ and the associated error term for $\cos x$. Hence, find the set of values of x for which $\cos x$ may be replaced by $1 - \frac{1}{2}x^2$ with an error magnitude no greater than $.5 \times 10^{-4}$.

5 (i) Obtain near $x = 0$ the quadratic approximation $P_2(x)$ and the associated error term for the function $f(x) = (1 + x)^{-1/2}$.
(ii) For a) $x = 0.1$, and b) $x = -0.1$, evaluate $P_2(x)$, and give an upper estimate on the error term.

(iii) In both cases a) and b) compare the difference between $P_2(x)$ and the value of $f(x)$ computed by your calculator, with your estimated error term.

6 (i) Obtain the degree three approximation $P_3(x)$ near $x = 2$, and the associated error term for the function $f(x) = 1/x$.
(ii) For a) $x = 1.9$, and b) $x = 2.01$, evaluate $P_3(x)$, and give an upper estimate on the error term.
(iii) In both cases a) and b) compare the difference between $P_3(x)$ and the value of $f(x)$ computed by your calculator, with your estimated error term.

7. Put

$$f(x) = \frac{1}{x} - \frac{1}{\sin x}$$

over a common denominator and then use
 (i) l'Hopital's rule
 (ii) Taylor's series for $\sin x$
to evaluate $\lim_{x\to 0} f(x)$. You should, of course, get the same answer in both cases.

8. Use series and a calculator to evaluate

$$\int_0^{0.1} e^{-x^2}\, dx$$

as accurately as your calculator will allow.

9. Work out the third order approximation $P_3(y)$ to $(1-y)^{-1/2}$ at $y = 0$ and use this, and your calculator, to give an approximation to

$$\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}.$$

Also, work out this integral exactly and compare your answer with the approximation.

10(i) Let

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Work out $f'(x)$ for $x \neq 0$, and differentiate from first principles to show that $f'(0) = 0$.
(ii) Show by induction on n that, for all integers $n > 0$, there is a polynomial $Q_n(y)$ such that, for $x \neq 0$,

$$\frac{d^n}{dx^n} f(x) = Q_n(1/x)e^{-1/x^2}.$$

(iii) Guess the value of $f^{(n)}(0)$ for all $n > 0$ and give any justification that you can for your answer.

11. Given that $g(x) = e^x \cos(x + \alpha)$, where α is a real constant, show that

$$\frac{dg}{dx} = \sqrt{2} e^x \cos(x + \alpha + \frac{\pi}{4}).$$

Now show that, for all integers $n \geq 0$,

$$\frac{d^n g}{dx^n} = (\sqrt{2})^n e^x \cos(x + \alpha + \frac{n\pi}{4}).$$

Find the Maclaurin series for $g(x)$ up to, and including, the term in x^3.

12. Solve each of the following equations, and sketch some solutions in the (x, y)-plane, marking clearly any solutions which are straight lines.

(i) $\frac{dy}{dx} = \frac{1}{y}$.

(ii) $\frac{dy}{dx} = xy$.

13.(i) Find all solutions of $y\frac{dy}{dx} + 2x = 0$ and sketch some solutions in the (x, y)-plane.

(ii) Find all solutions of $y\frac{dy}{dx} - \cos x = 0$. Using the Taylor series expansion of $\sin x$ about $\frac{\pi}{2}$, show that there are some solution curves of te form

$$y^2 + (x - \frac{\pi}{2})^2 - \frac{1}{12}(x - \frac{\pi}{2})^4 \cdots = \varepsilon$$

for small $\varepsilon > 0$. Show also, using the Taylor seies expansion of $(1 + t)^{\frac{1}{2}}$ that there are solution curves of the form

$$y = C + \frac{1}{C}\sin x - \frac{1}{2C^3}\sin^2 x \cdots$$

for $|C| > \sqrt{2}$.

14. Solve the following differential equations, subject to the given boundary conditions

$$\begin{array}{lll}
(i) & ye^{-2x}\frac{dy}{dx} = e^{-y^2}, & y(2) = 2; \\
(ii) & \frac{dy}{dx} = (x+1)y, & y(1) = 1; \\
(iii) & x + 1 + y\frac{dy}{dx} = 0, & y(0) = 1; \\
(iv) & (x+1)\frac{dy}{dx} = 2(1 + y^2), & y(0) = 1; \\
(v)) & \cot x\frac{dy}{dx} = 1 + y, & y(\pi/4) = 2.
\end{array}$$

15. Consider the model for the spread of a disease, where, in a population of N individuals, the number of infectives is I, and the differential equation for I is

$$\frac{dI}{dt} = \beta I(N - I),$$

where $\beta > 0$ is the transmission constant, and t is time. Show that if there is just one infective at $t = 0$, the number at time t is

$$I = \frac{N}{1 + (N-1)e^{-N\beta t}}.$$

What happens as $t \to \infty$?

16. Solve the following differential equations, subject to the given boundary conditions:

$$\begin{array}{lll} (i) & \dfrac{dy}{dx} = x - 2y, & y(0) = 1; \\ (ii) & x\dfrac{dy}{dx} - 2y = x^2, & y(1) = 0; \\ (iii) & x\dfrac{dy}{dx} + 2y = e^{2x}, & y(1) = 1; \\ (iv) & (1 + x^2)\dfrac{dy}{dx} - 2xy = 1 + x^2 & y(0) = 1. \end{array}$$

17. Find the general solution of the differential equation

$$(x-1)^3\frac{dy}{dx} + 4(x-1)^2 y = x + 1.$$

18. Find the solution of the differential equation

$$x\frac{dy}{dx} = 2y + x^3 \sec x \tan x$$

for which $y(\pi/3) = 2$.

19. Find the general solutions of the equations:

$$\begin{array}{ll} (i) & (x+1)\dfrac{dy}{dx} - 2x^2y - 2xy = e^{x^2}; \\ (ii) & \dfrac{dy}{dx} + 2x = 2xy. \end{array}$$

20. Find the general solutions of the following differential equations:

$$\begin{array}{ll} (i) & (x+y)\dfrac{dy}{dx} + x - y = 0; \\ (ii) & x\dfrac{dy}{dx} = xe^{y/x} + y; \\ (iii) & xy\dfrac{dy}{dx} = 3x^2 + y^2. \end{array}$$

21. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = x + 2.$$

22. Find the general solution of the equation

$$a_2\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_0y = 0$$

in each of the following cases:

$$\begin{array}{llll} a) & a_2 = 1, & a_1 = -1, & a_0 = -12, \\ b) & a_2 = 1, & a_1 = 2, & a_0 = 10, \\ c) & a_2 = 1, & a_1 = 8, & a_0 = 16, \\ d) & a_2 = 3, & a_1 = 7, & a_0 = 2, \\ e) & a_2 = 1, & a_1 = 6, & a_0 = 9. \end{array}$$

23. Find a general solution of the equation

$$a_2\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_0y = 0,$$

in each of the following cases:

$$\begin{array}{ll} a) & a_2 = 1,\ a_1 = 5,\ a_0 = 6, \\ b) & a_2 = 1,\ a_1 = -6,\ a_0 = 9, \\ c) & a_2 = 1,\ a_1 = 4,\ a_0 = 20, \\ d) & a_2 = 1,\ a_1 = 10,\ a_0 = 25. \end{array}$$

In each of cases a) and c), find the solution that satisifes the boundary conditions $y(0) = 0$ and $y'(0) = 2$.

24. Given the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = f(x),$$

find the complementary function.
Find also a particular integral, and the general solution, in each of the following cases:
(a) $f(x) = 6$,
(b) $f(x) = 10e^{3x}$,
(c) $f(x) = 4x - 2$,
(d) $f(x) = 20\sin x$.

25. Solve the differential equation

$$a_2\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_0y = f(x),$$

with $y = b_0$ and $\dfrac{dy}{dx} = b_1$ when $x = 0$, in each of the following cases:

(a) $a_2 = 1,\ a_1 = 1,\ a_0 = -6,\ f(x) = 8e^x,\ b_0 = 3,\ b_1 = -2.$

(b) $a_2 = 1,\ a_1 = 4,\ a_0 = 3,\ f(x) = 65\sin(2x),\ b_0 = -5,\ b_1 = -7.$

26. Solve the differential equations

$$\begin{array}{lll} (i) & \dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = 6x^2 + 5x + 3 & y(0) = 0, y'(0) = 0; \\ (ii) & \dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 2y = \sin x & y(0) = 0, y'(0) = 0. \end{array}$$

27. The equation governing a damped forced simple harmonic motion is

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + a^2k^2y = b\sin\omega t,$$

where k is a damping coefficient, a, b and ω are positive constants, $y = y(t)$ is the displacement and t is the time. Show that the particular integral for this differential equation may be written in the form

$$y_p = \frac{b}{R}\sin(\omega t - \phi),$$

where $R = ((a^2k^2 - \omega^2)^2 + 4\omega^2k^2)^{1/2}$ and ϕ is a constant angle to be determined.

28. Find the domain and range for each of the following functions:
(i) $f(x, y) = x^2 + y^2 + 1$,
(ii) $f(x, y) = x^2 - y^2 - 1$,
(iii) $f(x, y) = \ln(xy)$,
(iv) $f(x, y) = \dfrac{1}{x^2 + y^2 + xy}$. For this one, can you write $x^2 + xy + y^2$ as a sum of two squares? That will help to give a nice precise answer.

29. Sketch some level curves of the following functions, distinguishing between the cases $f(x, y) = c$ for $c = 0$, $c > 0$ and $c < 0$ (although in one case there is no level curve with $c < 0$).
(i) $f(x, y) = 4x^2 + y^2$.
(ii) $f(x, y) = y^2 + x$.
(iii) $f(x, y) = x^2 - 4y^2$.
30. By considering different paths of approach, show that the following functios have no limit as $(x, y) \to (0, 0)$.

$$(i)\ \frac{x^2 - y^2}{x^2 + y^2} \quad (ii)\ \frac{x^4}{x^4 + y}, \quad (iii)\ \frac{xy}{|xy|}.$$

31. Determine for each of the following functions whether it extends continuously to $(0, 0)$. If it does, give the value of the continuous extension function at

$(0,0)$.

$$(i) \quad \frac{x^2-y^2}{x^2+y^2}, \qquad (ii) \quad \frac{xy-x-y}{x+y},$$
$$(iii) \quad \frac{x^3-y^3}{x^2+y^2}, \qquad (iv) \quad \frac{\sin(x^2+y^2)}{x^2+y^2}.$$

32. Find $\partial f/\partial x$ and $\partial f/\partial y$ for each of the following:

$$(a) \quad f(x,y) = x^2 - xy + y^2, \qquad (b) \quad f(x,y) = (2x-3y)^3.$$

33. Find $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial z$ for each of the following:

$$(a) \quad f(x,y,z) = (x^2+y^2+z^2)^{-1/2}, \qquad (b)* \quad f(x,y,z) = yz\ln(xy).$$

34. Let $f(x,y) = \dfrac{x^2+y^2}{x+y}$ for $(x,y) \neq (0,0)$.
Find an expression for $\dfrac{\partial f}{\partial x}$.
Find the limit of $\dfrac{\partial f}{\partial x}$ as (x,y) tends to $(0,0)$ along each of the paths with equations $y=0, x=0$ and $y=x$.

35. Find all second order partial drivatives of

$$(a) \quad f(x,y) = \sin(xy), \qquad (b) \quad f(x,y) = xe^y + y + 1.$$

36. Show that the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ (c constant) is satisfied by

$$(a) \quad u(x,t) = x^3 - 3cx^2t + 3c^2xt^2 - c^3t^3,$$
$$(b) \quad u(x,t) = \sin(x+ct) + 2\cos(x-ct).$$

37. A solid, with rectangular sides, has length l, width w and height h so that its volume $V = lwh$. Given that l, w, h change by $\delta l, \delta w, \delta h$, use partial derivatives to estimate (in terms of $l, w, h, \delta l, \delta w, \delta h$) the change, δV, in V.
A hollow rectangular box, whose *inside* measurements are 5 m long, 3 m wide and 2 m deep, is made of wood 1 cm thick and the box has no top.

Use the previous result to estimate the volume of wood used. Find also the exact volume of wood used.

38. The area of a triangle is $(1/2)ab\sin C$, where a and b are the lengths of two of the sides and C is the size of the included angle. In surveying a triangular plot a, b and C are measured to be 150 m, 200 m and $\pi/3$ radians respectively. Estimate by how much the area calculation may be in error when

errors in both a and b have magnitude 0.25 m and the error in C has magnitude 0.04 radians.

Also estimate the percentage error in the area calculation.

39. In each of the following cases, express $\dfrac{dw}{dt}$ as a function of x, y (and maybe z) and t, by using the Chain Rule, and then evaluate $\dfrac{dw}{dt}$ at the given value of t:

a) $w(x,y) = x^2 + y^2$, $x = \cos t + \sin t$, $y = \cos t - \sin t$, $t = 0$;
b) $w(x,y,z) = z - \sin(xy)$, $x = t$, $y = \ln t$, $z = e^{t-1}$, $t = 1$.

40. Find $\dfrac{\partial z}{\partial u}$ when $u = 0$, $v = 1$, given that $z = \sin(xy) + x \sin y$, $x = u^2 + v^2$, $y = uv$.

41. Let $z = x^3 \sin[\pi y^2]$, where $x = u + v$ and $y = u - v$.
Use the chain rule to find $\dfrac{\partial z}{\partial v}$ when $u = 2$ and $v = 1$.

42. A rectangular box has sides of length a, b and c. These lengths are changing with time t. At $t = 0$, $a = 13$cm, $b = 9$cm, $c = 5$cm,

$$\frac{da}{dt} = \frac{dc}{dt} = 2\text{cm/sec and } \frac{db}{dt} = -5\text{cm/sec.}$$

Work out $\dfrac{dV}{dt}$ and $\dfrac{dS}{dt}$ at $t = 0$, where $V(a,b,c) = abc$ is the volume and $S(a,b,c) = 2(ab+bc+ca)$ is the surface area. Also, by computing $\dfrac{d(D^2)}{dt}$, where D is the diagonal of the box, determine whether the diagonal is increasing or decreasing in length at time $t = 0$.

43. Find the linear approximation to $f(x,y) = (x+y+2)^2$ at
a) $(x,y) = (0,0)$ and b) $(x,y) = (1,2)$.

44. Find the linear approximation to $f(x,y,z) = x^2 + y^2 + z^2$ at
a) $(x,y,z) = (1,1,1)$ and b) $(x,y,z) = (1,2,3)$.

45. Find the gradient of f at the point P, and the directional derivative of f at the point P in the direction of the vector $\mathbf{a}$, for:

a) $f(x,y,z) = xy + yz + zx$, $P = (1,-1,2)$, $\mathbf{a} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$;

b) $f(x,y,z) = 3e^x \cos(yz)$, $P = (0,0,0)$, $\mathbf{a} = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$;

c) $f(x,y,z) = \cos(xy) + e^{yz} + \ln(zx)$, $P = (1, 0, \frac{1}{2})$, $\mathbf{a} = \mathbf{i} + \mathbf{j} - \sqrt{2}\mathbf{k}$.

46. Find the direction in which f increases most rapidly at the point P.

Then find the derivative of f in that direction.

(a) $f(x, y) = x^{2/3} + xy + y^{2/3}$, $P = (-1, 1)$.

(b) $f(x, y, z) = \ln(xy) + \ln(yz) + \ln(zx)$, $P = (1, 1, 1)$.

47. Find, at the point P, equations for the tangent plane and the normal line to the surface with equation $f(x, y, z) = 0$.

(a) $f(x, y, z) = 2z - x^2$, $P = (2, 0, 2)$.

(b) $f(x, y, z) = x^2 + y^2 - 2xy - x + 3y - z + 4$, $P = (2, -3, 18)$.

48. Find a cartesian equation of the tangent plane to the surface with equation

$$f(x, y, z) = x^2y + y^2z + z^2x = 1$$

at the point with coordinates $(1, -1, 1)$.

49. Find parametric equations for the line that passes through a point P and is tangential to the curve of intersection of the surfaces with equations $f(x, y, z) = 0$ and $g(x, y, z) = 0$.

(a) $f(x, y, z) = xyz - 1$, $g(x, y, z) = x^2 + 2y^2 + 3z^2 - 6$, $P = (1, 1, 1)$.

(b) $f(x, y, z) = x^2 + y^2 - 4$, $g(x, y, z) = x^2 + y^2 - z$, $P(\sqrt{2}, \sqrt{2}, 4)$.

50. The derivative of $f(x, y)$ at $P = (1, 2)$ in the direction of $(\mathbf{i} + \mathbf{j})$ is $2\sqrt{2}$ and in the direction of $-2\mathbf{j}$ it is -3.
Find the derivative of f at P in the direction of $-\mathbf{i} - 2\mathbf{j}$.

51. Find all maxima, mimima and saddle points of each of the following functions. Also determine which, if any, of the maxima and minima are absolute.

a) $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$.

b) $f(x, y) = x^2 - 4xy + y^2 + 6y + 2$.

c) $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$.

d) $f(x, y) = 1/(x^2 + y^2 - 1)$.

e) $f(x, y) = xy + 2x - \ln(x^2y)$.

f) $f(x, y) = x^{-1} + xy + y^{-1}$.

52. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate, including the boundary where $x^2 + y^2 = 1$, is heated so that the temperature $T°C$ at the point (x, y) is given by

$$T = x^2 + 2y^2 - x.$$

Find the temperatures, and the coordinates, at the hottest and coldest points of the plate.

53. Find the absolute maxima and minima of the function

$$f(x, y) = x^2 - xy + y^2 + 1$$

on the domain given by the closed triangular region in the first quadrant bounded by the lines with equations $x = 0, y = 4$ and $y = x$.

54. Find the absolute maxima and minima of the function

$$f(x, y) = -2x^2 + 6xy - 3y$$

on the domain given by the closed triangular region in the first quadrant bounded by the lines with equations $y = 0$, $x = 1$ and $y = x$.

55. Find the shortest distance from the origin to the curve with equation $x^2y = 2$.

56. Find the shortest distance from the origin to the surface with equation $z = xy + 1$.

57. Find the maximum and minimum distances from the origin to the curve with equation

$$g(x, y) = x^2 + y^2 - 2x - 4y = 0.$$

58. Find the maximum distance from $(0, 0)$ to the ellipse

$$g(x, y) = 5x^2 + 4xy + 2y^2 = 2.$$

Hint: consider the square of the distance $f(x, y) = x^2 + y^2$.

59. A rectangle with sides parallel to the coordinate axes is inscribed in the ellipse with equation $x^2/16 + y^2/9 = 1$. Find the dimensions of the rectangle when it has maximum area.

60. You are asked to design a storage tank for liquid gas. The customer's specifications are for a cylindrical tank with hemispherical ends, and the tank is to hold 8000 m^3 of liquid. The customer also wishes to use the smallest possible amount of material in building the tank. Determine the recommended radius and height of the cylindrical portion of the tank.

61. Use Taylor's formula for $f(x, y)$ at the origin to find a quadratic polynomial approximation of f near the origin for $f(x, y) = e^x \ln(1 + y)$.

62. Use Taylor's formula for $f(x, y)$ at the origin to find a quadratic approximation to f near the origin for $f(x, y) = 1/(1 - x - y + xy)$.

63. Given that $f(x, y) = \dfrac{1}{7 - 3x + 2y - xy}$ find a quadratic approximation

to $f(x, y)$ which is valid near $(x, y) = (2, -3)$.

64. Evaluate the double integral $\int_R y dy dx$, where R is the region in the top right quadrant bounded by the straight lines $x = 0$ $y = 1$ and the curve $y = x^2$ confirm your result by reversing the order of integration.

65. Evaluate the repeated integral

$$\int_0^1 \int_x^1 \sin\left(\frac{\pi x}{y}\right) dy dx$$

by first changing the order of integration.

66. Find the volume of the region that lies under the surface with equation $z = x + y$ and above the triangle in the $x - y$ plane which is enclosed by the lines with equations $y = x$, $x = 0$ and $x + y = 2$.

67. Show that the area of the finite region in the first quadrant bounded by the parabola with equation $y = 6x - x^2$ and the straight line wth equation $y = x$, is $125/6$ square units. Find the centroid of the region. (The centroid is another word for centre of mass, with uniform density of mass.)

68. Find the centroid of the finite region in the top right quadrant bounded by the parabola with equation $y^2 = 2x$ and the straight lines with equations $y = 0$ and $x = 2$.

69 Find the centre of mass of the triangle T bounded by $x = 1$, $y = 0$ and $y = x$, where the density is $\rho(x, y) = x$.

70. Derive the formula for the area of a circle by evaulating the double integral $\int_R dx dy$, where R is the region inside the circle of radius a centred at the origin, by using polar coordinates r, θ.

71. Change the following cartesian repeated integral into an quivalent polar integral, and hence evaluate it:

$$\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2 + y^2) dy dx.$$

72.

(a) Solve $u = x - y, v = 2x + y$ for x and y in terms of u and v.
Then find the Jacobian $\partial(x, y)/\partial(u, v)$.

(b) Let R be the finite region in the first quadrant bounded by the lines with equations $y = 4 - 2x, y = 7 - 2x, y = x - 2, y = x + 1$. By changing the variables with the equations in (a) and integrating over a region in the (u, v) plane, evaluate

$$\int_R (2x^2 - xy - y^2) dS.$$

73. (a) By changing the order of integration, evaluate

$$\int_0^1 \int_{x^4}^1 \frac{x^7}{1+y^3}\, dydx.$$

(b) Sketch the region S in the Oxy plane specified by

$$0 \leq x \leq \sqrt{3},\ 0 \leq y \leq x\sqrt{3}.$$

By transforming to polar coordinates, evaluate

$$\int\int_S \frac{1}{\sqrt{(x^2+y^2)}}\, dxdy.$$

You may assume that the integral of $\sec\theta$ is $\ln(\sec\theta + \tan\theta)$

Brief Syllabus of the Module

Taylor polynomials and Taylor series, Taylor's Theorem

Ordinary Differential equations. Separation of variables. Integral curves. The integrating factor method for linear first order differential equations. Reduction to separale form for homogeneous equations. Initial conditions. Linear second order ODE's with constant coefficients. Particular solutions and complementary functions. Simple harmonic motion.

Functions of several variables. Domain, range, limits, continuity. Partial derivatives. Linearisation. The chain rule, implicit differentiation. Directional derivatives, gradients of functions, tangent planes. Maxima, minima, saddles for functions of two variables. Constrained extrema. Lagrange multipliers. Taylor series in several variables.

Double integrals over regions in the plane. Change of order of integration. Area. Centroid and centre of mass of a 2-dimensional body. Change of variable. Jacobians. Double integration in polar coordinates.

Recommended Text

Thomas' Calculus updated 10th edition. Finnney, Weir, Giordano, Addison Wesley £47 . Most people who want it probably already have it, as it was the recommended text for MATH101. There are 5 copies in Blackwells on 8.1.07 The 11th or international editions are fine.

Probably everyone has the university calculator by now. (This is the only model allowed in exams.) The Sharp EL-531WB-WH is available in the Guild Shop.