Selective Variational Image Segmentation Combined with Registration: models and algorithms

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***Abstract*—In this paper, I present some new and joint work on local and selective segmentation models and algorithms which have potential applications in medical imaging. First I review a familiar segmentation model of global energy minimization framework in two dimensions (three dimensions may be presented similarly). Then I discuss selective segmentation models and several refined models where pre-defined geometric constraints guide local segmentation. Such 2D models can be generalized to 3D and some brief experiments are given to demonstrate the ideas of the paper. Finally I discuss the use of image registration methods to obtain geometric constraints or equivalent initial contours towards an automatic segmentation framework.**

As mentioned, the work discussed here represents a small portion of results obtained in the Liverpool's Centre for Mathematical Imaging Techniques (CMIT) and is jointly carried out with collaborators; for this paper, these include Noor Badshah (Peshawar, Pakistan), Jian-ping Zhang and Bo Yu (Dalian, China), Lavdie Rada (Liverpool), Noppadol Chumchob (Silpakorn, Thailand), Carlos Brito (Yucatan, Mexico), and Derek A. Gould (Royal Liverpool University Hospital, Liverpool).

I. Introduction

Image segmentation is an important task in a number of real life image processing applications. Although edge based models have been in wide use for much longer time, region based models offer more robust methods for many challenging problems. Following the seminal works of Osher-Sethian (1988) [25], Mumford-Shah (1989) [24] and Chan-Vese (2001) [14], more and more image segmentation models are proposed, refined and tested. However almost all these models aim to identify all edges and features in an image and such global models may not be needed in some applications where extraction of an particular feature is intended.

Nevertheless, variational segmentation methods using global energy optimization, due to their robustness and reliability, are increasingly used to detect objects in recent years. These techniques might be subdivided in three major categories: (i) the edge detector based contour methods [3], [2], [10], [11], [12], [18], [20], (ii) the region based methods [4], [5], [6], [26], [14], [21], [24], [28], [31] and (iii) combined methods using the above two approaches.

For a comprehensive survey of the literature, we refer the reader to the more recent books by [3], [13], [27], [22], [30] and the references therein.

II. Global energy minimization models

As a special case of the Mumford and Shah segmentation technique [24], the Chan-Vese variational model of active contours without edges [14] has gained much popularity in the community due to its simplicity. It has been used successfully for segmentation of all global features of an image. Since this model does not use directly the gradient of the image as a stopping process, it is extremely robust to presence of noise as a region based method.

Without loss of generality, we only briefly review this model. Assume that a given image z may be approximated by two regions of piecewise constant intensities, of distinct values $z_i = z_{in}$ and $z_o = z_{outside}$. Thus the object to be detected is automatically represented by the region with intensities closest to the value z_i. Let Γ denote the boundary that separates the domain Ω into two regions Ω_1 (the feature) and $\Omega_2 = \Omega\backslash\overline{\Omega_1}$ (the background). Then $z \approx z_i$ inside the object (inside Γ) and $z \approx z_o$ outside the object (outside Γ). Precisely, Chan and Vese proposed the variational problem [14]

$$\min_{c_1,c_2,\Gamma} J(\Gamma, c_1, c_2) \tag{1}$$

for the segmentation of all image features, where

$$J(\Gamma, c_1, c_2) = \mu \text{length}(\Gamma) + \lambda_1 \int_{\text{inside}(\Gamma)} |z(x,y) - c_1|^2 d\Omega$$
$$+\lambda_2 \int_{\text{outside}(\Gamma)} |z(x,y) - c_2|^2 d\Omega. \tag{2}$$

Here $z(x,y)$ is the original image, c_1 and c_2 are the average values of z inside and outside of the variable contour Γ, also μ, λ_1 and λ_2 are non-negative fixed parameters that should be related to the features' diameter. As both the integral and the limits of integration in equation (2) are not known, to overcome this problem, a level set function ϕ (by Osher and Sethian [25]) is introduced. The unknown curve Γ can be represented by the zero level set of Lipschitz function $\phi : \Omega \to \mathbb{R}$ such that

$$\begin{cases} \text{inside}(\Gamma) = \ \Omega_1 \ = \{(x,y) \in \Omega & \Big| \ \phi(x,y) > 0\}, \\ \text{outside}(\Gamma) = \ \Omega_2 \ = \{(x,y) \in \Omega & \Big| \ \phi(x,y) < 0\}, \\ \Gamma = \partial\Omega_1 = \{(x,y) \in \Omega & \Big| \ \phi(x,y) = 0\}. \end{cases}$$

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Define the Heaviside and the Dirac delta function as

$$H(x) = \begin{cases} 1 \text{ if } x \geq 0 \\ 0 \text{ if } x < 0 \end{cases} \qquad \text{and} \qquad \delta(x) = H'(x),$$

and, given ϕ as above, equation (2) is rewritten in the following way (with $F_1 = |z(x,y) - c_1|^2$)

$$J(\phi, c_1, c_2) = \mu \int_\Omega |\nabla H(\phi)| d\Omega + \lambda_1 \int_\Omega F_1 H(\phi(x,y)) d\Omega$$
$$+\lambda_2 \int_\Omega |z(x,y) - c_2|^2 (1 - H(\phi(x,y))) d\Omega. \qquad (3)$$

Once the level set function ϕ is obtained, the segmented image is given by

$$u = c_1 H(\phi) + c_2(1 - H(\phi)).$$

To minimize J with respect to c_1, c_2, keeping $\phi(x,y)$ fixed, we have

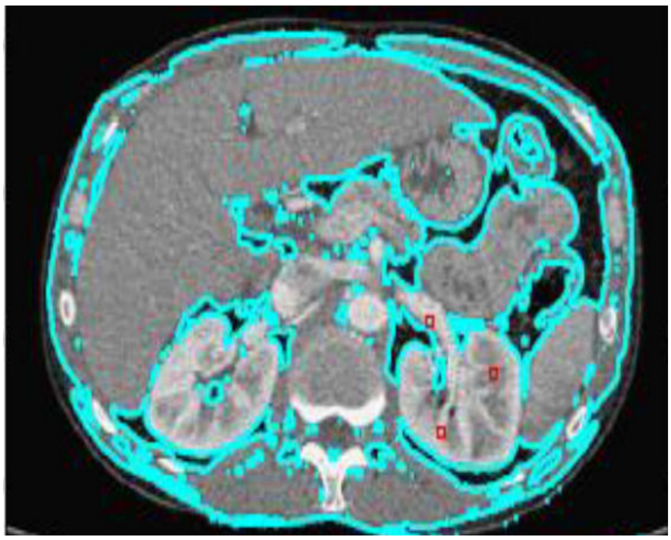
$$c_1(\phi(x,y)) = \frac{\int_\Omega z(x,y) H(\phi(x,y)) d\Omega}{\int_\Omega H(\phi(x,y)) d\Omega} \qquad (4)$$

if $\int_\Omega H(\phi(x,y)) d\Omega > 0$ (i.e the curve has a nonempty interior in Ω), and

$$c_2(\phi(x,y)) = \frac{\int_\Omega z(x,y)(1 - H(\phi(x,y))) d\Omega}{\int_\Omega (1 - H(\phi(x,y))) d\Omega}, \qquad (5)$$

if $\int_\Omega (1 - H(\phi(x,y))) dx > 0$ (i.e the the curve has a nonempty exterior in Ω). A typical segmented result may be shown in Fig.1, where one sees that the two-phase method does its job nicely but such a result is hardly of any use in medical imaging.

Fig. 1: The result of a typical global segmentation model



III. New local energy minimization models

Aiming to locate a particular feature only, one desires to have a model that does the selective segmentation job of a region growing method such as [1] but is more robust and reliable. Recent work by Gout and Guyader [19] and Badshah-Chen [6] proposed two different variational models for selective segmentation. The Gout-Guyader model [19] is based on edge information of the object while the Badshah-Chen model [6] combines an edge based model with region based information. Both models are useful and can segment a range of images, but there are cases which appear too challenging for either model. The latter model, with the help of region information, improved the former in robustness and segmentation quality in case of noisy images. It should be remarked that for global segmentation, the idea of combing an edge based model with region based information was earlier used in [8], [29] among other works which involve combined models.

Below is the Badshah-Chen model [6]:

$$\min_{\phi(x,y), c_1, c_2} J(\phi(x,y), c_1, c_2) =$$
$$\mu \int_\Omega d(x,y) g(|\nabla z(x,y)|) |\nabla H(\phi(x,y))| d\Omega$$
$$+ \lambda_1 \int_\Omega |z(x,y) - c_1|^2 H(\phi(x,y)) d\Omega$$
$$+ \lambda_2 \int_\Omega |z(x,y) - c_2|^2 (1 - H(\phi(x,y))) d\Omega, \qquad (6)$$

where $d = d(x,y)$ is the minimal distance function from all given geometric constraints which roughly point out where the desired feature is located, and g is an edge detection function. The main problem with (6) lies in the last term which helps to reach out to all background features or push redundant features to domain Ω_1, potentially failing the model.

Several ways have been proposed to overcome this problem. In [26], we propose to replace this term by using a second global level set function in a dual level set framework ($\phi_L = \phi_{Local}, \phi_G = \phi_{Global}$):

$$\begin{aligned} \min_{\Gamma_L, \Gamma_G, c_1, c_2} & J(\Gamma_L, \Gamma_G, c_1, c_2) = \\ & \mu_1 \int_{\Gamma_L} d(x,y) g(|\nabla z(x,y)|) ds + \\ & \mu_2 \int_{\Gamma_G} g(|\nabla z(x,y)|) ds + \\ & \lambda_{1G} \int_{\text{inside}(\Gamma_G)} |z(x,y) - c_1|^2 d\Omega + \\ & \lambda_{2G} \int_{\text{outside}(\Gamma_G)} |z(x,y) - c_2|^2 d\Omega + \\ & \lambda_1 \int_{\text{inside}(\Gamma_L)} |z(x,y) - c_1|^2 d\Omega + \\ & \lambda_2 \int_{\text{outside}(\Gamma_L) \cap \text{inside}(\Gamma_G)} |z(x,y) - c_1|^2 d\Omega + \\ & \lambda_3 \int_{\text{outside}(\Gamma_L) \cap \text{outside}(\Gamma_G)} |z(x,y) - c_2|^2 d\Omega, \end{aligned} \qquad (7)$$

where the desirable feature is contained in $\Omega_{1,L}$ or inside(Γ_L).

A second method for improving (6) is to replace the L_2 fitting of Mumford-Shah or Chan-Vese type by a coefficient of variation fitting term as in [7]:

$$J(c_1, c_2, \Gamma) = \mu \int_\Gamma d(x,y) g(|\nabla z|) ds \quad + \tag{8}$$

$$\lambda_1 \int_{outside(\Gamma)} \frac{|z - c_1|^2}{c_1^2} d\Omega + \lambda_2 \int_{inside(\Gamma)} \frac{|z - c_2|^2}{c_2^2} d\Omega,$$

where the idea is to promote local solutions near the initial contour that is set by the geometric constraints.

The third method is to replace the whole domain Ω by a local dynamic domain for both fitting terms in (6). Then the evolving curve will not move away to nearby features to lead to redundant segmentation. The precise formulation is the following [32]:

$$\min J(\Gamma, c_1, c_2) = \int_\Gamma d(x,y) g(|\nabla z|) ds \quad + \tag{9}$$

$$\lambda_1 \int_{\Omega_{in,\gamma}(\Gamma)} (z - c_1)^2 d\Omega + \lambda_2 \int_{\Omega_{out,\gamma}(\Gamma)} (z - c_2)^2 d\Omega,$$

where $\Omega_{in,\gamma}(\Gamma)$ denotes a γ band domain away from Γ, instead the whole feature domain Ω_1. Assume that ϕ is positive inside the desired region and negative outside it. Then the local fitting energy function

$$b(\phi(x), \gamma) = H(\phi(x) - \gamma)(1 - H(\phi(x) + \gamma)) \tag{10}$$

characterizes the domain $\Omega_\gamma = \Omega_{in,\gamma}(\Gamma) \cup \Gamma \cup \Omega_{out,\gamma}(\Gamma)$ which is a narrow band region surrounding the local boundary Γ. The band size may be even made variable before each iteration step to achieve a robust model.

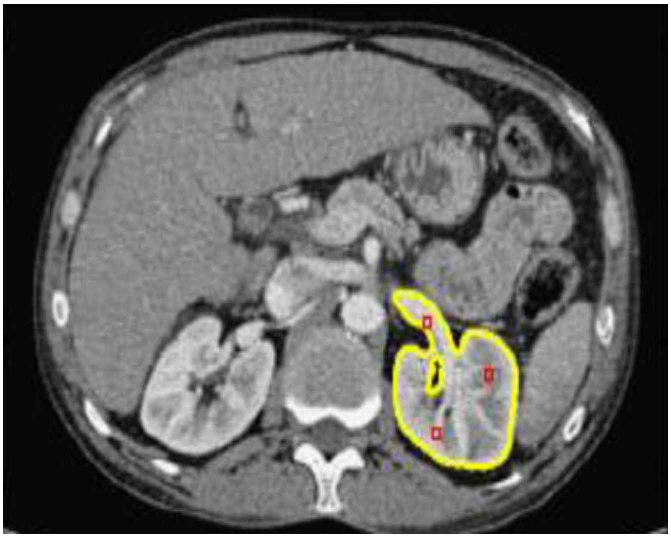
Other approaches are also in progress. With any of these models, an excellent segmentation result in Fig. 2 can be achieved for the problem in Fig.1. Here one observes that the desirable feature is isolated without involving nearby features. In fact generalization to the three dimensions is feasible and results such as in Fig.3 have been achieved. One often obtains results such as from Fig.3 by building up a sequence of 2D segmented slices. Here they can be from a 3D model directly.

IV. Use of Registration Models for Robustness

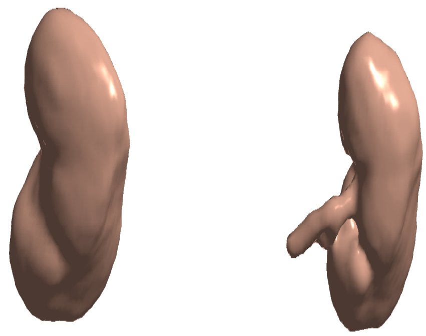
A careful reader can see from Figs 1-2 three red dots which are the geometric constraints given to define d and the initial level set functions. If one has defined such positions before for one image, to segment a new image of a similar nature (e.g. both of livers in medical imaging), it is natural to re-use the previous knowledge of such constraints on the new image through a registration process of the two images.

Suppose that two images R (the reference) and T (the template), intended for registration, are given as the continuous functions mapping from an image domain $\Omega \subset \mathbb{R}^2$ into $V = [a,b] \subset \mathbb{R}_0^+$ and each component u_j of $\boldsymbol{u}$ is the function of the spatial position $\mathbf{x} = (x_1, x_2)^\top \in \Omega$. The

Fig. 2: The result of local segmentation models



s



following variational model with a new similarity functional and regularization terms is proposed [16]

$$\min_{\boldsymbol{u},c} \mathcal{J}_{\alpha_1,\alpha_2}(\boldsymbol{u}, c) = \mathcal{D}(\boldsymbol{u}, c) + \alpha_1 \mathcal{R}_1(\boldsymbol{u}) + \alpha_2 \mathcal{R}_2(c) \tag{11}$$

with

$$\mathcal{D}(\boldsymbol{u}, c) = \frac{1}{2} \int_\Omega \left(c(\mathbf{x}) T(\mathbf{x} + \boldsymbol{u}(\mathbf{x})) - R(\mathbf{x})\right)^2 d\mathbf{x},$$

$$\mathcal{R}_1(\boldsymbol{u}) = \sum_{l=1}^{2} \int_\Omega \Phi(\kappa(u_l)) d\mathbf{x},$$

$$\mathcal{R}_2(c) = \mathcal{K}(c) = \int_\Omega \Phi(\kappa(c)) d\mathbf{x},$$

where $\Phi(x) = x^2$. It should be remarked that much work on using a mean curvature stems from our success of [9] in solving the curvature equation (or fourth order and highly nonlinear partial differential equations) efficiently.

Although there exist many models of variational framework [23], for mono-modality images, we recommend the model of [17]; for multi-modality images, we believe the new model by [16] is much better in terms of robustness than widely used mutual information based models.

Finally for both segmentation and registration models, one may follow the simple algorithms from [15] to develop fast multigrid algorithms [17], [16], [4], [5].

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