LOCAL IMAGE FEATURES RESULTING FROM 3-DIMENSIONAL GEOMETRIC FEATURES, ILLUMINATION, AND MOVEMENT: I

J. DAMON[1 2], P. GIBLIN[1], AND G. HASLINGER[1]

Introduction

For objects in images which are illuminated by a single light source, our visual system is able to take advantage of a number of visual clues involving the interaction of the geometric features of the objects, the shade/shadow regions on the objects, and the (apparent) contours resulting from viewer direction to differentiate between objects and determine their shapes and positions. Typically the clues are obtained from the local configurations resulting from the interaction of one or more of these ingredients. Furthermore, these configurations may change as a result of movement of light source(s), the objects in the image (including change in geometric shape as e.g. resulting from human movement), and viewer movement. Then, there are the following general goals.

General Goals :

(1) Understand what types of local configurations of geometric features, shade/(cast) shadow curves, and apparent contours "we expect to see" in a static image and relate these local configurations to the underlying 3-dimensional shape and positions of objects in an image.

(2) When there is movement for one or more of these ingredients, the configurations will undergo a number of changes. Determine the changes in terms of basic "generic transitions" in local configuration structure and relate them to 3-dimensional structure and position.

In this paper we shall achieve the first goal, and establish the second for the important case where viewer movement occurs. In fact, the ability of a viewer to integrate the local clues to distinguish objects and their shapes involves a use of small movement in the viewer direction to decide among ambiguities in the local clues. We will determine the local possible "generic transitions" when the light source and objects are fixed.

Specifically, we will give the classification of the local configurations for the case of a fixed single light source and objects having generic local geometric features. The classification includes both the stable views, in which the configurations do not change under small viewer movement, and the generic transitions in local configurations, which a viewer can expect to see under viewer movement. The classification is obtained by applying a rigorous mathematical analysis using methods from singularity theory; the full mathematical details are contained in [DGH].

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A very early work which considers the interpretation of line drawings as 3D objects is that of Clowes [Cl]; for other early references see the book by Horn [H]. In the book by Sugihara [S] single-view line drawings of polyhedral objects are considered. Many more recent authors are concerned with line drawings, interpreting them as 3D objects from single views, for example see the work of Varley and others, [VSM].

Refinements of these results are for: transitions of apparent contours for smooth surfaces due to Koenderink, and van Doorn[KvD], Arnold [A1], and Gaffney-Ruas [Gaf]; polygonal surfaces with shade and movement Huffman [Hu], Mackworth [Mkw], line drawings with several geometric features but without shade/shadow nor movement Malik [Mlk], piecewise smooth surfaces, Rieger [Ri], aspect graphs of curved surfaces and polyhedral surfaces, Petitjean, Ponce, and Kriegman [PPK], [KrP], Kanizsa figures, Cassalles et al [CCM] [CCM2], and apparent contours for smooth surfaces with shade/shadow, Demazure, and Henry-Merle, [DHM] [HM], Donati-Stolfi [DS], and Donati [Di] (where we correct a number of aspects of the classification given there) and preliminary work of Fitzgerald [F].

In this work we shall subsume (and in several cases complete or correct) these earlier special cases which involve apparent contours without geometric features or without shade/shadows. However, we emphasize that we concentrate on the local structure and local transitions, and do not consider global aspect graphs as were considered in several of the above works.

We consider the interaction of three ingredients: the geometric features of surfaces, shade curves and cast shadow curves, and the apparent contours resulting from viewer direction. The objects in the image are fixed in position with generic geometric features. The precise assumptions about the boundary surfaces of objects in the image, their possible geometric features (such as edges, corners, etc.), and the fixed single light source will be explained in §1. Several of the many possibilities for the generic interactions between shade/shadow curves, geometric features, and apparent contours are illustrated in the images in figure 1.

For a collection of objects with generic geometric features, "almost all" choices of light direction will yield resulting configurations of shade/shadow curves with the geometric features which are "stable", i.e. they do not change their form under small changes in light direction (here we are not yet speaking of views of these configurations). After an arbitrarily small change, any light direction will have this property, and we suppose our light direction does. We consider the situation where the fixed light direction and object positions are such that the interaction of shade/shadow curves (S) and geometric features (F) are generic (i.e. stable under small movement of light direction).

Then, using singularity theory, we first give the classification of the possible stable configurations of shade/shadow curves by themselves (S) and their interaction with geometric features (SF). These all give rise to stable views by taking "regular images" of these configurations (for (S) see figure 9, for (SF) see figures 10 for edges, figure 12 for creases, and figures 15 and 16 for corners). Here "regular images" means each surface and curve is projected diffeomorphically onto its image. For just geometric features, we first consider the case of "uniform light/shade", by which we mean near a point of interest each sheet of the surface is either entirely in light or shade, so no additional geometric information is gained from shade/shadow

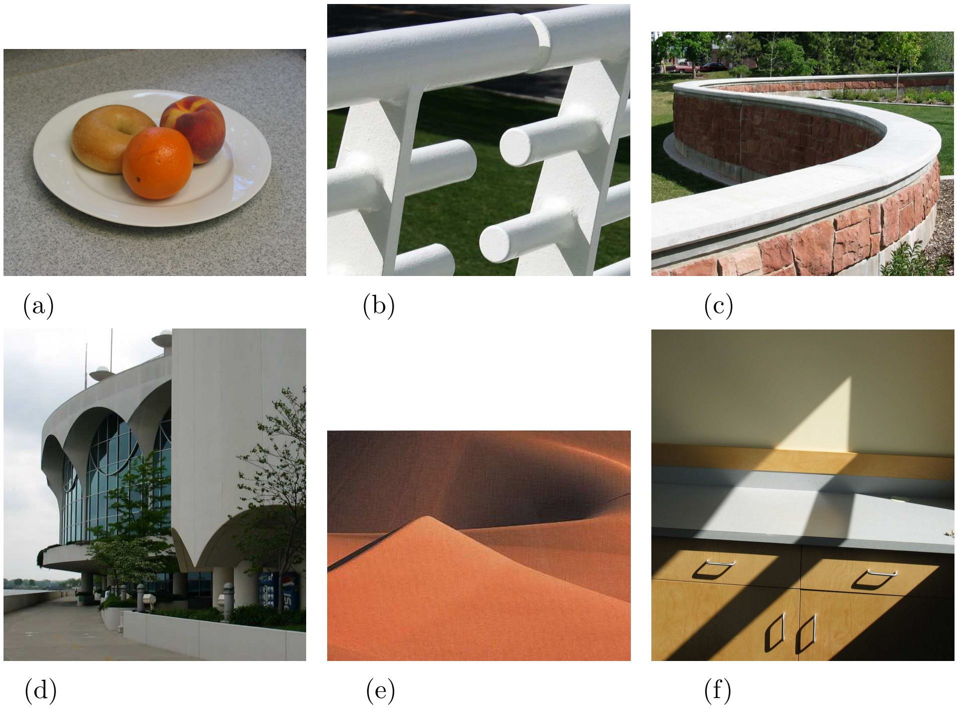

FIGURE 1. Images exhibiting creases, corners, marking curves and shade/shadow curves. Interactions involve: (a) apparent contours and shade/shadow; (b) shade/shadow curves with creases; (c) crease, contours, and shade/shadows; (d) shade/shadow curves with corners and creases; (e) shade/shadow curves with contours and creases; and (f) cast shadow curves with creases and marking curves.

curves. Again regular images of such configurations (F) give the next class of stable views (see figures 10, 11, figures 14, 15, and 16 for corners).

Then, in addition to (S),(F), (SF), the complete classification involves the interaction of apparent contours (C) with the stable shade/shadow–geometric feature configurations, in all possible additional combinations (C) (SC), (FC), (SFC) (summarized by figure 2). Taken together the classifications for each of the seven configuration possibilities give all possible stable configurations. These classifications are given in (3.1), including the remaining possibilities shown: for (C) and (SC) see figure 9, for (FC) (with uniform shade light) see figure 13. Surprisingly there are no stable interactions of all three ingredients (SFC), but these do occur prominently in the generic transitions for viewer movement.

Fourthly, we determine the multilocal stable configurations of these ingredients. This means that local features from different objects or different parts of the same object interact. The interaction can occur from either occlusion, allowing as possibilities occluding edges, ridge creases, and apparent contours, or from cast shadows from a distance. The shadow is either cast by a geometric feature, or there is

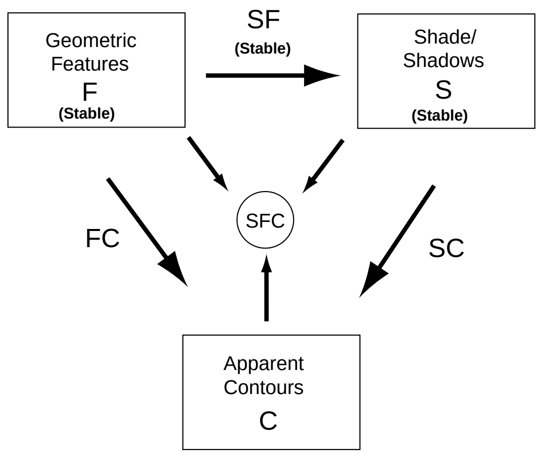

FIGURE 2. The possible local interactions between shade/shadow curves (S), geometric features (F), and apparent contours (C).

a smooth cast shadow curve which intersects an edge, ridge crease, or apparent contour.

These many possibilities can be concisely summarized by the resulting configuration of curves representing the shade/shadow curves, the curves representing geometric features, crease, edge, and marking curves, and curves representing apparent contours. We divide these curves into two types: "hard curves" and "soft curves". The soft curves consist of the shade/shadow curves, and are not sharply defined curves but must be detected using a Canny-type edge method based on intensity change. The hard edges are all of the remaining curves for geometric features and apparent contours. The entire classification of stable local and multilocal interactions is summarized by the "alphabet" of nineteen stable curve configurations given in figure 3. These include the sixteen curve configurations corresponding to local stable interactions explained in figures 18 and 19, and five curve configurations corresponding to multilocal stable interactions in figures 20, with two overlapping cases.

Second, considerably more information about object shape and position can be deduced from the generic changes occurring in the configurations as a result of small changes in viewer direction. In our case, three of the seven possibilities for configurations, (S), (F), and (SF) are stable under small viewer movement. However, changes in viewer direction can cause transitions in the remaining configurations (C), (SC), (CF), and (SCF). We will provide a complete classification of generic transitions for each of these configuration types, making use of a combination of earlier abstract classifications from singularity theory and extending them as appropriate. In this first paper, we give an overview of the general classification in (5.1). We give the specific classifications for (C), (SC), and (CF) (excluding transitions for corners) in (5.4) and (5.6). The classifications of transitions for: corners, for interactions of all three ingredients (SFC), and for the multilocal cases will be given in Part II [DGH2].

To be precise, when we speak of classifying the stable configurations and the generic changes under viewer movement, we mean allowing "equivalence up to

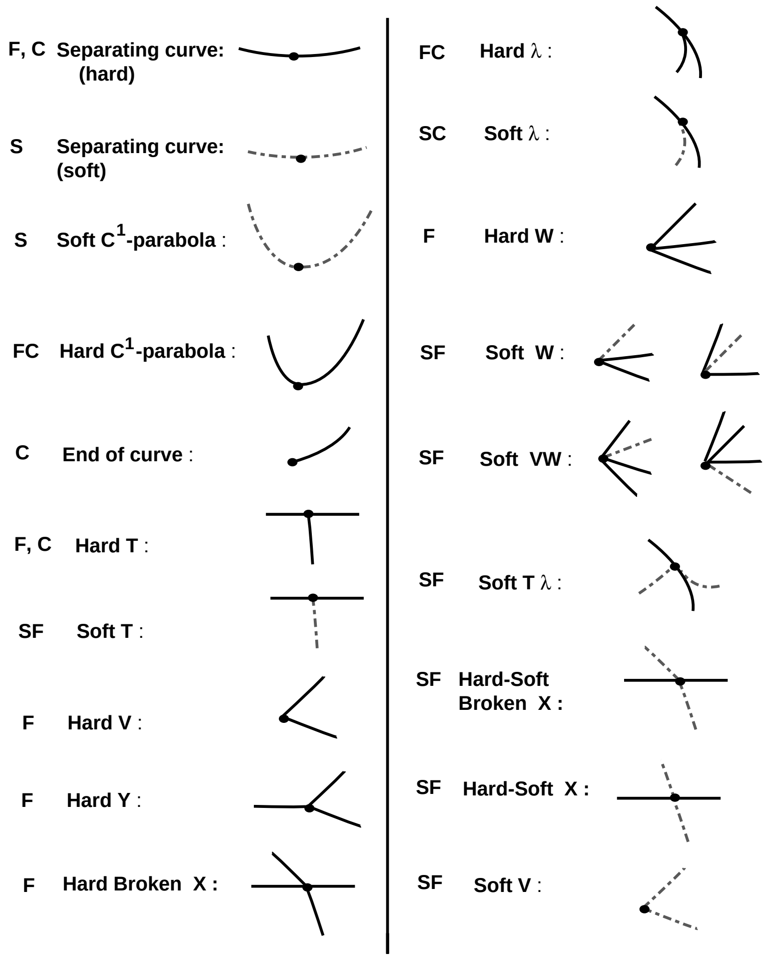

FIGURE 3. Classification of the "alphabet" of possible configurations of hard curves (solid black) and soft curves (dashed grey) corresponding to the local configurations of features-shade/shadow-apparent contours resulting from the classification and the mutilocal classification . Also indicated are the configuration-types based on figure 2.

applying local diffeomorphisms " which preserve the geometric features and the shade/shadow curves. These classifications are consequences of the mathematical theorems proven in [DGH]. We briefly explain in §2 how the methods of singularity theory allow us to carry out the classifications. For an interested reader who does not wish to read the full mathematical treatment in [DGH], a more detailed explanation of the specific singularity theoretic methods will be given to Part II [DGH2]. We shall further explain the relation of our classification with the earlier work on these questions in the appropriate sections.

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1. Viewing Illuminated Surfaces with Geometric Features

We begin by explaining what exactly we assume about the surfaces, their geometric features, and the light sources.producing shade and shadows.

Lighting and Viewer Direction. We allow multiple light sources; however, we suppose that all but one source contributes only as diffuse background light, and one light source is principally responsible for the shade and cast shadows. We suppose that this light source is fixed and sufficiently distant that the light rays are parallel and from a single direction. When we use equations to describe the surface, we will assume that the light rays are parallel to the y–axis and come from the direction of the positive y–axis.

Likewise we consider the case where the viewing direction is not in direct line with the light direction. Because of the properties of the shade/shadow, geometric features and apparent contours, we may, after a linear change of coordinates, suppose that the viewing direction is along the x-axis from the positive x–direction. We suppose that the view is along parallel light rays from the object, as opposed to central projection. It will follow from the genericity properties of the configurations that this restriction does not alter the local configuration properties that we obtain.

Shade/Shadows and Specularity. We will only consider perfectly diffuse surfaces, for which incident light is reflected equally in all directions. This class includes Lambertian surfaces. These surfaces do not have specular highlights. Because the nature of the specular highlights depend on both the material of the surface and upon the viewing direction (in particular the BRDF function of the material, see e.g. Koenderink-Pont [KP] or in the book of Horn-Brooks [HB]), we shall ignore specular effects at this time in order to not overly complicate the discussion.

The *shade curves* on a surface theoretically arise from where the light rays meet the surface tangentially. However, in fact, on shade curves there is a gradual transition from light to dark. In reality, the shade curve lies in this band where we go from light to dark. Such a curve is a “soft curve” and is more precisely captured by a type of Canny-edges method applied to intensity.

In addition, there are also *cast shadow curves* where these tangential rays continue until they meet the surface again. Although these cast shadow curves will be sharper than the shade curves, they still are subject to diffraction. and so are not as sharp as curves defining various geometric features. For this reason we will also refer to them as “soft curves”. For local configurations, we consider local cast shadows. Cast shadows from a distance are included among the multilocal configurations; and, in fact, the interaction with hard edges (edges, ridge creases, and apparent contours) of cast shadows from distant objects behave like marking curves.

Geometric Features. We allow the boundary surfaces of objects in the image to have geometric features which may consist of: creases (either ridges or valleys), corners (of various types), boundary edges (as for thin surfaces such as sheets of paper or leaves), and marking curves. Marking curves will generally mean either actual curves, or implied curves such as separating regions with e.g. different color or texture. We understand each of these as local features, even though the local features usually fit together to give global features.

We explain what exactly we mean by these local geometric features. In each case, the general form of the feature is obtained from a standard model for the feature by applying a local diffeomorphism of $\mathbb{R}^3$ but otherwise are not restricted. Then, diffeomorphisms applied to these standard models allow geometric features with curved surfaces and curves (see figures 4, 5, and 6). .

The features and their local models are given as follows.

(1) A *(boundary) edge* of a surface is modeled by the boundary edge of a half plane;
(2) A (*ridge or valley*) *crease* is modeled by a standard crease formed from two half planes in the x-z and y-z planes joined along the z–axis, which is their common edge. The union of these half planes divides $\mathbb{R}^3$ into two regions as shown in figure 4. Each choice of region corresponds to the two possibilities of ridge or valley creases.
(3) A *corner* is modeled by the union of parts of the x-z, y-z, and x-y planes with edges on the x, y, and z axes and which divides $\mathbb{R}^3$ into two regions, as in figure 5. There are four essentially distinct possibilities which we list as convex, concave, saddle and notch corners as shown in figure 6.
(4) A *marking curve* can be on a smooth surface, a surface with edge, or a surface with crease, where the marking curve is on just one sheet of the crease or cuts across the crease and is on both sheets. Depending on the case, the marking curve is either modeled by a line in a plane, or by a line on the model for an edge or crease as shown in figure 7.

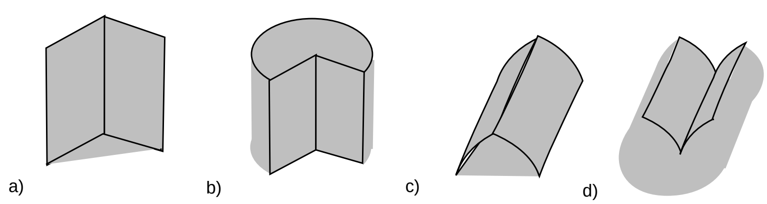


FIGURE 4. Models for creases and the general curved versions: a) and c) ridge creases, and b) and d) furrow (valley) creases.

We summarize the descriptions of the different geometric features in Table 1.

Genericity of Shade/Shadow. We suppose that the objects in the image are in fixed positions with generic geometric features as just described. Since the light direction is also fixed, the relation between the shade/shadow curves and the geometric features will also be fixed. For a collection of objects with generic geometric features, “almost all” choices of light direction will yield resulting configurations of shade/shadow curves with the geometric features which “are stable”, i.e. they do not change their form under small changes in light direction (here we are not yet speaking of views of these configurations). This statement has a well-defined mathematical sense and this is a consequence of a mathematical result of John Mather [Ma2]. We briefly explain the underling mathematics in §2.

After an arbitrarily small change, any light direction will have this property, and we suppose our light direction does. We consider the situation where the fixed light

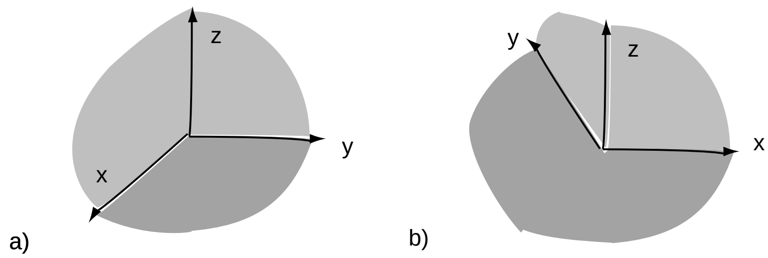


FIGURE 5. Models for corners: a) is the model for convex and concave corners, depending on whether the region is within the first octant or the complement; b) is the model for saddle and notch corners, depending on whether the region in behind and below or in front and above the model faces.

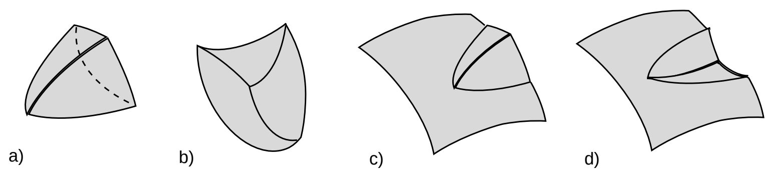


FIGURE 6. General curved versions of corners: a) is a convex corner; b) is a concave corner; c) is a saddle corner; and d) is a notch corner.

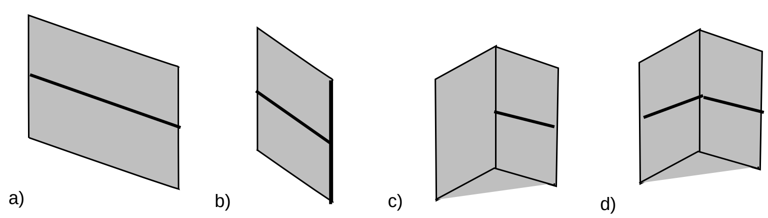


FIGURE 7. Models for marking curve: a) on smooth surface; b) at edge point of surface; c) at crease point, on one sheet; and d) at crease point, on both sheets. There are also versions of c) and d) for valley creases.

direction and object positions are such that the interaction of shade/shadow curves (S) and geometric features (F) are generic (i.e. stable under small movement of light direction).

Multilocal Considerations. Multilocal configurations of these ingredients occur when local features from different objects or widely separated parts of the same object interact. The interaction can occur from either occlusion or cast shadows.

Because the light source and object are fixed, we assume the cast shadow from a distance meets any other geometric feature generically, which means it intersects any geometric curve non-tangentially and does not intersect isolated points such as corner points. If the shadow is cast by a geometric feature such as a V point (see

Geometric Feature	**Definition**	**Physical Examples**
Ridge crease	Outward curve along which two surfaces meet transversely	Edges of tables, bookcases, etc; the sharp arête-like ridges formed on sand dunes
Furrow (valley) crease	Inward curve along which two surfaces meet transversely	The join between two pages of an open book
Boundary edge	Points on the boundary of a surface	Edges of knife blade, leaf, sheet of paper
Marking curve	Differentiable curves on a smooth surface or surface with edge or crease. See figure 7	Stripes on an animal (e.g. Zebra); curves delineating sharp changes in texture or color on a surface
Corners *Convex corner*	The meeting point of three edges (i.e. where three smooth surfaces meet at a point) figures 5 a) and 6 b)	Four different cases: Corners of tables, bookcases etc.
Concave corner	figures 5 a) and 6 a)	Corners of a room, or inside a box.
Saddle corner	figures 5 b) and 6 c)	The corner at point where dorsal fin of a shark meets its body, the inner corner at a level join of two pieces of wood
Notch corner	figures 5 b) and 6 d)	Corner of a closed mouth

TABLE 1. Geometric features of surfaces.

figure 3), the cast shadow of the vertex of the V lies in a smooth part of a surface, that is, not on a crease or edge curve, nor on a marking curve.

For occlusion, any geometric curve can be occluded, including edges, ridge creases, and apparent contours, with the occluding surface being bounded at the occlusion point by an edge, ridge crease, or an apparent contour.

We do include the stable curve configurations for the multilocal case so that we provide in one location Fig. 3 the complete classification of stable curve configurations (which consists of both local and multilocal configurations, see Figs. 18 and 19, and Fig. 20 for the multi-local cases). However, the detailed analysis of the multilocal case will be postponed until part II of this paper.

Other Considerations. We have avoided a number of considerations which would have allowed an essentially unending series of complexities. We have stayed with geometric features that are "generic". Thus, generically, exactly three surfaces meet at a corner. However, certain constructions, such as e.g. church steeples, do not follow this. Likewise, marking curves on surfaces may meet in special ways as a result of designs. Consequently, what we consider is not meant to be all inclusive but rather only includes generic features. However, most objects in images do have generic properties.

Second, the light source is fixed; as a result only certain configurations of shade/shadow curves occur generically. If we allow the light source to move, then for fixed generic viewpoint, we can list the generic transitions in the configurations of geometric features and shade/shadow curves which occur. These differ from those changes resulting from change in viewpoint. However, they are derived from the list of transitions for (FC) by using the view direction as the light direction and appropriately giving the shade/cast shadow curves.

2. Explanation of How Singularity Theory Yields the Classifications

We briefly expain how the methods of singularity theory may be applied to obtain the classsifications of both the stable views and the generic transitions which occur for configurations of geometric features, shade/shadow curves, and apparent contours.

The method we use reduces the analysis of the interactions to the classification of view projection maps while preserving the configurations of geometric features and shade/shadow curves. This classification is actually for abstract smooth mappings while preserving the configurations as subsets. Then, it is determined whether cases of the classification can be realized by geometric projections with shade/shadow configurations as explained in §6.

Because the light source is fixed, we first classify the singularities of the light direction projection map to obtain the *stable configurations* of shade/shadow curves with geometric features of the object surface. Such configurations consist of specific configurations of curves on surfaces with geometric features. Second, we allow movement of the view direction. We classify the *stable view projection mappings*, for which the interaction of apparent contours with the stable configurations of shade-shadow and geometric features does not change under sufficiently small changes in view direction. We also classify the generic changes in interactions resulting from small movements of view direction involving no more than two parameters (corresponding to movement in the view sphere).

A basic contribution of our method is to show that the equivalences of abstract mappings whch preserve the stable configurations of curves on surfaces with geometric features satisfy a collection of mathematical properties which can be summarized by saying the equivalences form a "geometric subgroup of $\mathcal{A}$ or $\mathcal{K}$" as defined in [D1] (or [D1a]) and extended in [D2]. This implies that all of the basic theorems of singularity theory apply, allowing us to fully carry out the classifications. In carrying out these classifications, we are able to benefit from earlier classifications [BG2], [Ta1], and [Ta2] which have used special cases of this method of equivalence.

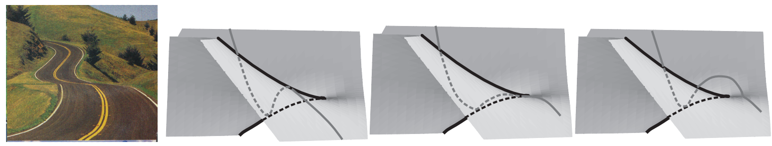
An alternate approach was proposed for studing the interaction of apparent contours with just shade-shadow by Demazure, Henry, Merle, [DHM], [HM] and Donati-Stolfi in [Di] and [DS]. They proposed classifying the pairs of mappings (view projection, light direction projection) from the smooth object surface. There is an intrinsic problem with this approach because such a pair forms a "divergent diagram" of mappings in the sense of Dufour [Du]. Although Dufour has classified in certain low dimension ranges the "stable singularities", the basic theorems of singularity theory do not apply to this case. Hence, the classification of generic transitions cannot be carried out; nor is there an extension to include geometric features.

Classifying Stable Interactions. First, the stable configurations of shade-shadow curves with geometric geometric features are determined. The equivalences of light direction projection mappings (i.e. projecting the surface onto the plane perpendicular to the light direction) using nonlinear changes of coordinates preserving the geometric features form a geometric subgroup of $\mathcal{A}$. Then, the "versal unfolding theorem" characterizes by an infinitesimal algebraic criterion the stable mappings under this equivalence. Furthermore, yet another theorem, the "finite determinacy theorem", provides models using polynomial equations. This provides the list of stable interactions of shade-shadow with geometric features. This list is obtained from Whitney's original classification of abstract stable mappings of surfaces to the plane [Wh], the stable mappings on surfaces with boundary due to Bruce-Giblin [BG2], applied as well to marking curves, and the stable mappings on surfaces with crease curves and corners Tari [Ta1], [Ta2], where we allow more possibilities for corners. These correspond to the "regular" stable configurations (F), (S), and (SF) given in (3.1). Already in the case of (SC) a new phenomenon occurs because regular projections of stable configurations involving shade-shadows and corners or creases are stable in a weaker "topological sense" where the equivalence is differentiable in the complement of the central point of interest (by results in [D2]).

Given this classification of stable shade-shadow-geometric feature configurations, we then classify the view projection mappings under the equivalences preserving each of these stable shade-shadow-geometric features configurations. By the explicit structure of the stable shade-shadow-features configurations, the equivalences of the view projection mappings preserving these configurations again form a geometric subgroup of $\mathcal{A}$, so all of the basic theorems of singularity again apply. We again use the infinitesimal algebraic criterion to classify the stable mappings, yielding the stable interactions of the apparent contours with the stable configurations (3.1) given for (C), (FC) (in the case of uniform lighting), (SC) and (SFC). Again for (C), (FC) this is a consequence of the classifications of Whitney and Tari. For (SC), we obtain an extra stable case given as figure 9 f). This was earlier obtained by Donati [Di] [DS]. Surprisingly, there are no stable (SFC) interactions; however, to conclude this requires the analysis given for the transitions in (SFC).

The multilocal classification is based on the specific local classifications and the multilocal equivalences again form geometric subgroups for which all of the basic theorems again apply. For completeness, the stable multilocal configurations are provided here by the "alphabet" of curve configurations given in figure 20. However, we postpone the analysis to obtain them along with their generic transitions until part II [DGH2].

Classifying Generic Transitions. Third, we consider transitions arising from movement of viewer direction (but with the light source and objects remaining fixed). In general, the changes which occur in the configurations as a result of viewer movement are given by a series of "generic transitions". We give a classification of these generic transitions. From a given viewing direction, there are two independent directions to move the view direction and observe changes in the local configuration (moving directly toward the point where the local view is directed does not change the local configuration). By a configuration being non-stable we mean that although the configuration of geometric features with the shade/shadow curves is stable, the view direction projection of this configuration changes under small movement of view direction. An example of this is illustrated in figure 8 Although there is a single image of the road, we see that for the parallel marking lines on the road, there is a transition involving breaking as the lines move from left to right. This



a) b) c) d)

FIGURE 8. **Semi-cusp Transition** (a): as the lines on the road move across they encounter a cusp of an apparent contour and the lines appear to break; this is equivalent to moving our view direction so the cusp moves across the road. In b) - d), we see the same transition which results from change of viewpoint of a marking curve for the "semi- cusp". In figure 26, we see a second version of the semi-cusp transition where the visible part of the marking curve points and the apparent contour at the cusp point form a C^1–parabola.

transition is modeled by one of the two semi-cusp transitions seen in b) - d) of the figure.

Methods of singularity theory for geometric subgroups allow us to both classify these unstable view direction projections, and to determine whether the small movement of viewer direction from the particular one will result in seeing all possible small changes in configuration. The mathematical terminology for this property of capturing all possible small changes by view direction movement is that change in viewer direction "provides a versal unfolding" of the unstable view projection map. There is a way to investigate properties of versal unfoldings without ever considering view specific projection maps, so we know in advance what properties to expect.

For the equivalence of abstract mappings preserving the stable configuration of shade-shadow curves and geometric features, there is a number which we can associate to the view projection map, its codimension, which indicates how many parameters are necessary to capture all possible small changes in the configuration, resulting from perturbations of the mapping (not necessarily given by movement of viewpoint). For example, the codimensions for the appropriate equivalences are denoted by $\mathcal{A}_e$-codim in Table 2, ${}_X\mathcal{A}_e$-codim in Table 3 , and $\mathcal{S}_e$-codim in Table 4. We need only classify those whose codimensions are at most two.

Codimension 1 *versus Codimension* 2 *Transitions.* There is a fundamental difference between the transitions for codimension 1 cases versus those of codimension 2. The viewsphere of possible viewing directions will contain curves representing codimension 1 transitions and these curves will meet or self-intersect in isolated points representing codimension 2 transitions.

In the codimension 1 case, changing the view direction so that such a curve is crossed at any nonzero angle will produce the transition, while moving the view direction along the curve will leave the view qualitatively unchanged. Thus, for the generic transitions which occur from the one-parameter movement of viewpoint (such as movement in time), only codimension one transitions will be seen generically. This is the predominant method of observing transitions, and we shall pay special attention to the codimension 1 transitions.

Surface $z = f(x, y)$	Name	$\mathcal{A}_e$-codim
x	regular map	0
x^2	fold	0
$x^3 + yx$	cusp	0
$x^3 + y^2x$	"lips"	1
$x^3 - y^2x$	"beaks"	1
$x^4 + yx$	swallowtail	1
$x^4 + yx^2 + y^2x$	beaks to swallowtail	2
$x^4 + y^3x$	"goose"	2
$x^5 + yx^3 + yx$	2 swallowtails	2
$x^5 - yx^3 + yx$	2 swallowtails	2

TABLE 2. Classification of view projections for a smooth surface up to $\mathcal{A}$–codimension 2. The equations are model equations for surfaces which when viewed from the initial view direction along the positive x–axis, exhibit the indicated behavior.

By contrast in the codimension 2 case, change can occur in different ways as the view direction moves away from the initial direction corresponding to a codimension 2 point. To describe all the changes which occur, a "clock diagram" is used to show configurations of features as the view direction moves in a small circle around its initial position. Because we typically consider transitions as a path in the viewsphere, such a path may pass near a codimension 2 transition point but generically not through it. Thus, the path will cut through a succession of different codimension 1 transitions which occur near the codimension 2 point. We will defer any detailed consideration of codimension 2 transitions until part II.

Generic Transitions for (C), (FC), and (SC). The classification of such "generic transitions" begins with classification of generic transitions for apparent contours for views of a smooth surface in uniform light under viewer movement. This was begun by Koenderink and Van Doorn [KvD] and completed independently by mathematicians V. Arnold [A1], and T.Gaffney and M. Ruas [Gaf] (also see [Gi]). Here the equivalence is called "$\mathcal{A}$–equivalence". The classification is given in table 2, where a surface is viewed from the positive x–axis, so the projection mapping of $z = f(x, y)$ sends (x, y) to $(f(x, y), y)$. Furthermore, for each of these types, viewer movement provides a versal unfolding, so all possible small changes are attained.

The classifications of Bruce-Giblin and Tari extend this classification to abstract mappings of codimension at most two for the equivalences preserving geometric features. The classification of generic projections require that we use these classifications, refining them to allow for visibility, other curves on surfaces, and additional types of corners. In addition, the classification again requires that we use the weaker topological equivalence to classify a number of the transitions. Together these give the generic transitions for (C) and (FC). Because of the large number of cases for codimension one transitions for corners, we present them in Part II.

For the configurations which involve both shade-shadow curves or these curves together with geometric features, the transitions are of type (SC) or (SFC). In these cases, we must expand the classifications to allow more general configurations of curves on surfaces possibly with creases and corners.

The *interactions of apparent contours with stable shade-shadow configurations* (SC) are obtained by classifying view projection mappings while preserving shade/shadow configurations ($\mathcal{S}$–equivalence). The classification of generic transitions is given by Table 4 and are explained in (5.4). These correct in a number of ways the list given by Donati [Di]. Furthermore, there is a new property which occurs here which does not occur earlier in the classification. Even if a configuration defined by an abstract mapping can be realized by a projection of a smooth surface with shade-shadow curves, it need not follow that the the transitions given by the versal unfolding can all be realized using only movement of view direction. In §6, we explain that there are geometric criteria for a smooth surface to exhibit specific shade-shadow and apparent contour configurations which prevent certain abstract transitions from occurring.

Common Models for Generic Transitions across Types. Furthermore, For the remaining three types of configurations (FC), (SC), and (SFC), there are quite a number of different stable configurations for geometric features or shade/shadow. In principle, a different classification of transitions of view projections would have to be carried out for each stable configuration. For practical purposes, this would quickly become both unmanageable and very unhelpful for computer imaging understanding. However, a basic principle is at work that allows certain basic abstract classifications to contribute in multiple repeated ways (with slight variations) to many of the classifications which occur.

For example, in table 2, those view projections of $\mathcal{A}_e$–codimension at most one. also contribute to the classifications which involve a stable configuration which is either: a smooth curve for a geometric feature, namely, edge curve, crease curve (where the singularity only occurs on one sheet) or marking curve, or a smooth shade/shadow curve. The classification of Bruce-Giblin [BG2] of singularities of abstract local mappings on the edge of a surface extends to the classification of local abstract mappings from the plane to itself, where a smooth curve in the source plane must be preserved (alternately see [Go]). This classification can be viewed as describing how the singularities in table 2 interact with the smooth curve, and how the curve itself projects onto the plane. It can be applied to any of the above configurations, provided we take into account visibility and whether we can realize the mapping as a projection of a surface having the given configuration. This turns out to be an issue only when the curve is a shadow curve; see §6. Then, the classification of codimension 1 generic transitions of types semi-cusp, semilips/semi-beaks, and boundary cusps are given by (4.1). These transitions occur for: edges, marking curves, a single sheet at a crease, and for "fold shade curves". These are listed in Table 3 along with the underlying apparent contour singularity from Table 2. Hence, a single classification for the view projection map is transformed into common classifications for multiple configurations.

To complete the classification of generic transitions for (SFC), we refine the classification of transitions for (FC) by adding the additional stable shade shadow curve configurations, and then refining the classifications to take into account the preservation of the additional curve configurations. We also postpone the details to part II.

3. **Classification of Stable Views of Configurations**

We give the classification of stable views involving the interaction of geometric features, shade/shadow, and apparent contour. The stable views involve both local and the multilocal stable configurations. The stable local configurations involve the seven types of interactions under the situations that we described in the previous section. Furthermore, we also give the mutlilocal classification which allows the interaction of different objects seen from a common viewpoint as a result of either occlusion or cast shadow from a distant object.

The classification we give subsumes, extends, and in some cases corrects, the partial classifications in the references given earlier.

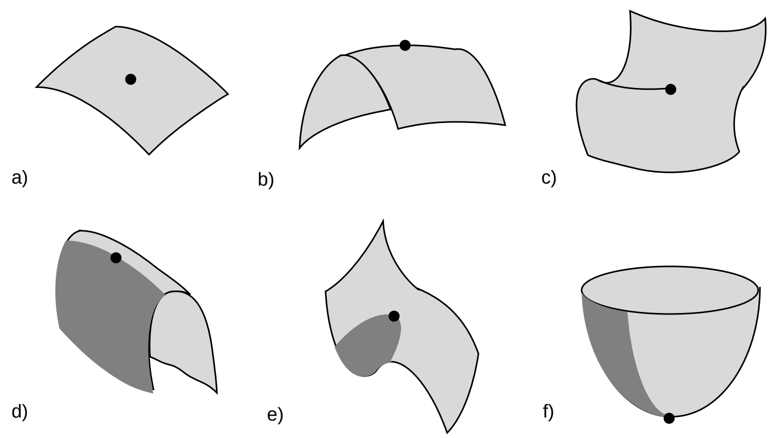


FIGURE 9. Stable apparent contours (C): a) regular image, with no apparent contour. b) fold , and c) cusp. Stable shade/shadow configurations (S): d) fold shade and e) cusp shade, and f) the one additional stable view of a fold shade curve with apparent contour (SC); this is called a *semifold*. For e), the shade/shadow curve consists of two pieces, a shade curve on the right and a cast shadow curve on the left, which meet to form a C^1 curve (a "C^1 parabola").

3.1 (**Stable Interactions for Shade/Shadow - Geometric Features - Apparent Contours**). *The stable interactions of geometric features, shade/shadows, and apparent contours are given by the local interactions and the multilocal interactions.*

The stable local interactions are given by the seven classes consisting of those involving either one ingredient: apparent contours (C), shade/shadow curves (S), or geometric features (F), or the interaction of two of these ingredients: shade/shadow and geometric features (SF); shade/shadow and apparent contours (SC); or geometric features and apparent contours (FC). The interaction of all three ingredients (SFC) does not occur stably. The following are the classifications for each type, where in the figures, the local configuration is around the blackened point.

(1) *The stable views for (C), (S), and (SC) are given by stable projections (regular, fold or cusp) in the light or view directions, with one case of fold projections for both. These are shown in figure 9.*

(2) *The stable views for (F) are regular projections of generic geometric features (edges, marking curves, creases, and corners) with uniform shade/light: for edges, figure 10 a) and b); for marking curves, figure 17 b) - i); for creases, figure 11; and for convex/concave corners, figures 14, for saddle corners, figure 15 a) - d), and notch corners, figure 16 a) - h).*
(3) *The stable views for (SF) are: for edges, figure 10 g) and h); for creases, figure 12; for saddle corners, figure 15 e) - g); and for notch corners, figure 16 i) - l).*
(4) *The stable views for (FC) with uniform shade light are : for edges, figure 10 c) - f); for marking curves, figure 17 a); and for creases, figure 13.*

Second, the stable multilocal classification consists of either an occlusion or a cast shadow from a distant object (or part of the object). The occlusion results from the partial occlusion of a marking, edge, crease, apparent contour, or shade/shadow curve by either a region of an object bounded at the occlusion point by an edge, ridge crease, or apparent contour. The cast shadow is either a smooth curve which cuts across a marking, edge, crease, apparent contour, or shade/shadow curve, or the cast shadow which forms a V on a smooth surface, with the shaded region filling the interior or exterior of the V.

Thirdly, for each stable view, the collection of curves forms a configuration which belongs to one of the cases in the "alphabet" of possibilities given in figures 18 and 19. The corresponding "alphabet" for the multi-local cases is given in figure 20.

Finally, there are specific mathematical conditions for each of the stable views which can be given in simplified form by the equations for the abstract mappings in the appropriate classifications, taking into account visibility. The full list is given in [DGH]*, and we illustrate several of the cases as the codimension* 0 *cases in Table 2 and Table 4.*

Remark 3.2. There are several Important consequences of this classification theorem for designing algorithms which detect features in natural images. These will be discussed in §7.

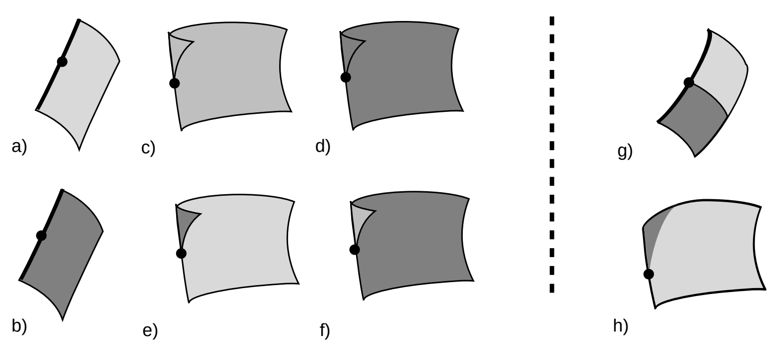


FIGURE 10. Edges: (F) a) and b); (FC): edges with apparent contours and uniform shade/light c) -f); and (SF) with shade/shadow curves: g) shade fold curve,and h) cast shadow curve.

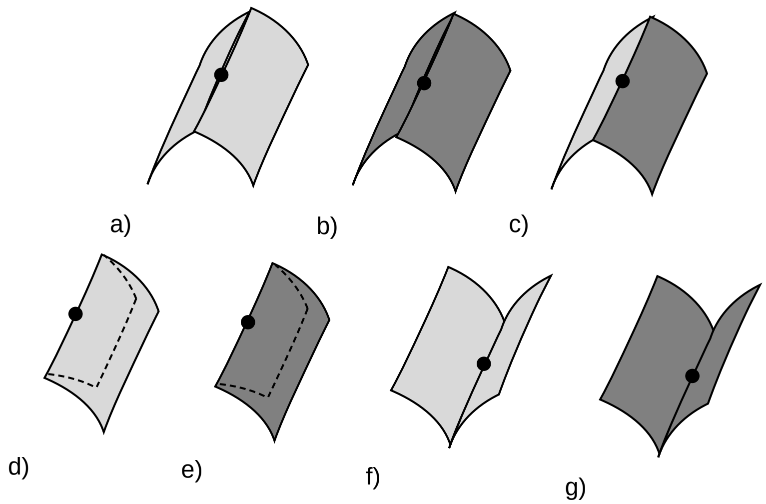


FIGURE 11. Creases with uniform shade/light (F)

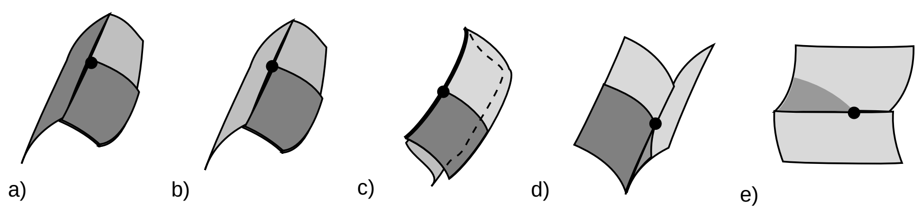


FIGURE 12. Creases (SF): creases with shade/shadow curves: a) and b) and c) shade fold curve; with c) one sheet visible; d) shade fold and cast shadow curve; and e) just cast shadow curve. There is also case f) where the sheet in e) without cast shadow is hidden and the image is the same as h) of Figure 10.

4. The Classification of Generic Transitions

The general results described in §2 are used to carry out the classification of generic transitions. The starting point is the classification of generic transitions for apparent contours under viewer movement given in table 2 which give the generic transitions for (C). We first give a partial classification of codimension 1 transitions (4.1) for (FC) and (SC) given by the transitions of type semi-cusp, semi-lips/semi-beaks, and boundary cusp for edges, marking curves, crease curves, and "fold shade curves". These are the common model transitions of codimension 1 in table 3, where each row represents instances of the single projection behavior for various meanings for the distinguished smooth curve.

4.1 (Codimension 1 Transitions in Table 3). *The feature curve is any of: a marking curve on smooth surface, a surface with a boundary edge or crease, or a surface with a (fold) shade curve. The transitions occur when the feature curve interacts with the contour generator, which is the curve on the surface that maps under view projection to the apparent contour as follows.*

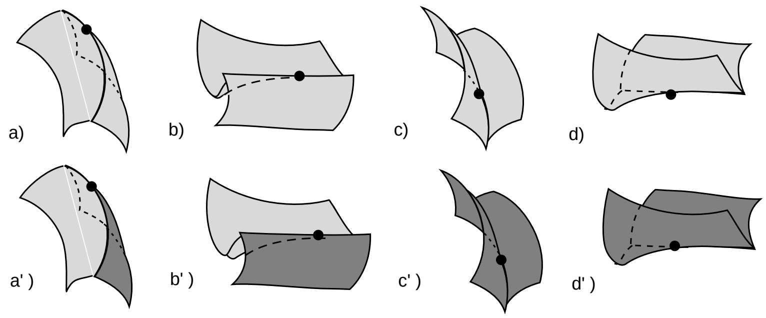


Figure 13. Creases with apparent contours (and uniform shade/light) (FC): a) visible crease and apparent contour; b) visible crease with invisible apparent contour; c) and d) partially visible crease with visible apparent contour - c) with both sheets visible or d) only one sheet visible.

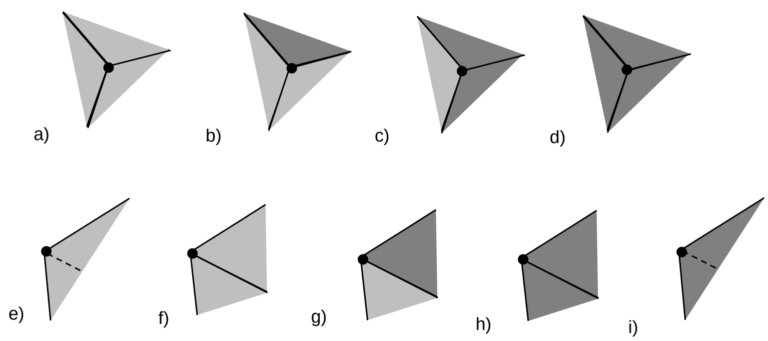


Figure 14. Convex and Concave Corners with uniform shade/light (F). Note that only a) and d) can occur for concave corners.

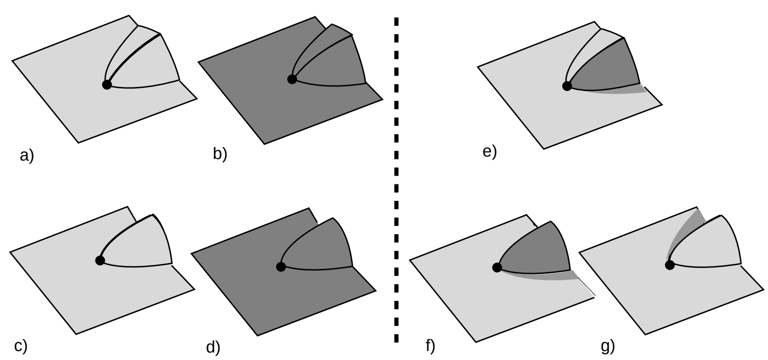


Figure 15. Saddle Corners: with uniform shade/light (F) a) - d); and with cast shadows (SF) e) - g).

(1) Semi-Cusp: *It occurs when the geometric feature curve and contour generator are* transverse *at the point but that the initial view direction is along the*

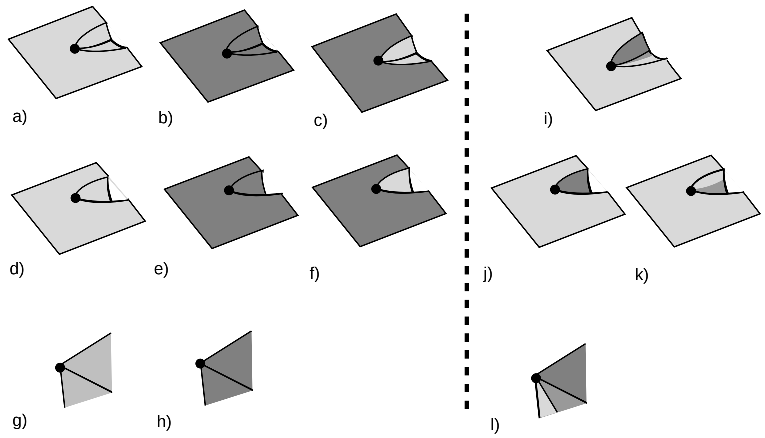


FIGURE 16. Notch Corners: with uniform shade/light (F) a) - h); and with cast shadows (SF) i) - l).

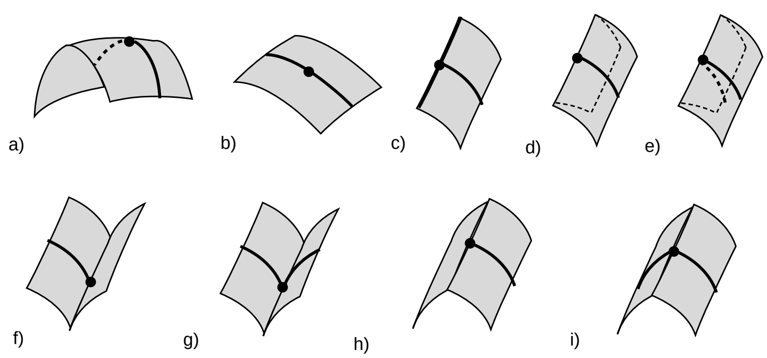


FIGURE 17. Marking Curves (with uniform shade/light, here shown with light) : a) fold apparent contour on smooth surface (FC) and the remaining without apparent contours (F): b) regular image on smooth surface; c) meeting edge curve of surface ; and meeting crease curve of surface, d), f) and h) on one sheet, and e), g) and i) on two sheets. There are also other uniform shade/light cases with some sheets in shade as in figures 10 a) and b) and 11.

tangent to the contour generator. In the initial view, the apparent contour has a cusp, and this cusp is tangent (at a 'C^1 parabola point' or a 'λ junction') to the feature. There are two visually distinct semi-cusp transitions depending on which side of the tangent to the apparent contour at the cusp point the geometric feature curve lies. Compare figures 8 and 26.

(2) Semi Lips/Beaks: *One of these occurs when a contour generator becomes tangent to the surface feature curve. The transition either causes a feature curve not intersecting the contour, to break after meeting it (Semi-Beaks); or an invisible part of the feature curve becomes visible along the contour (Semi-Lips). Examples are illustrated in figures 21, 22.*

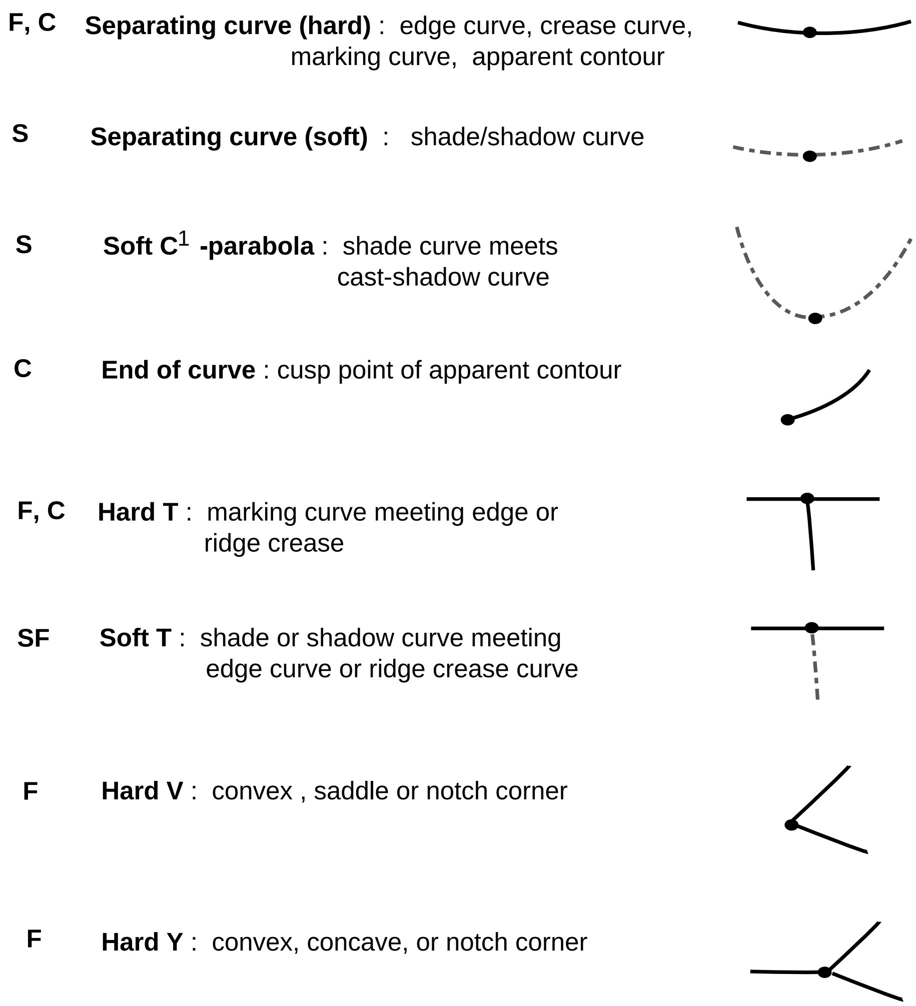


FIGURE 18. First part of the "alphabet" of possible configurations of hard curves (solid black) and soft curves (dashed grey) corresponding to the local configurations of features-shade/shadow-apparent contours resulting from the classification. Also indicated are the possible configurations yielding each curve configuration.

(3) Boundary Cusp: *It occurs when the apparent contour is a fold, hence appears smooth in the image, but the view projection mapping of the surface feature curve is singular (it exhibits a cusp). The transition changes the side that the apparent contour curve meets the feature curve in a semi-fold, see figures 23, 24, and 25.*

Notation : In the figures which follow, we indicate the stable types appearing under transitions, using the following *convention for junction points*: T-junction - □; cusp or endpoint - •; C^1-parabola point or a λ-junction - ∘.

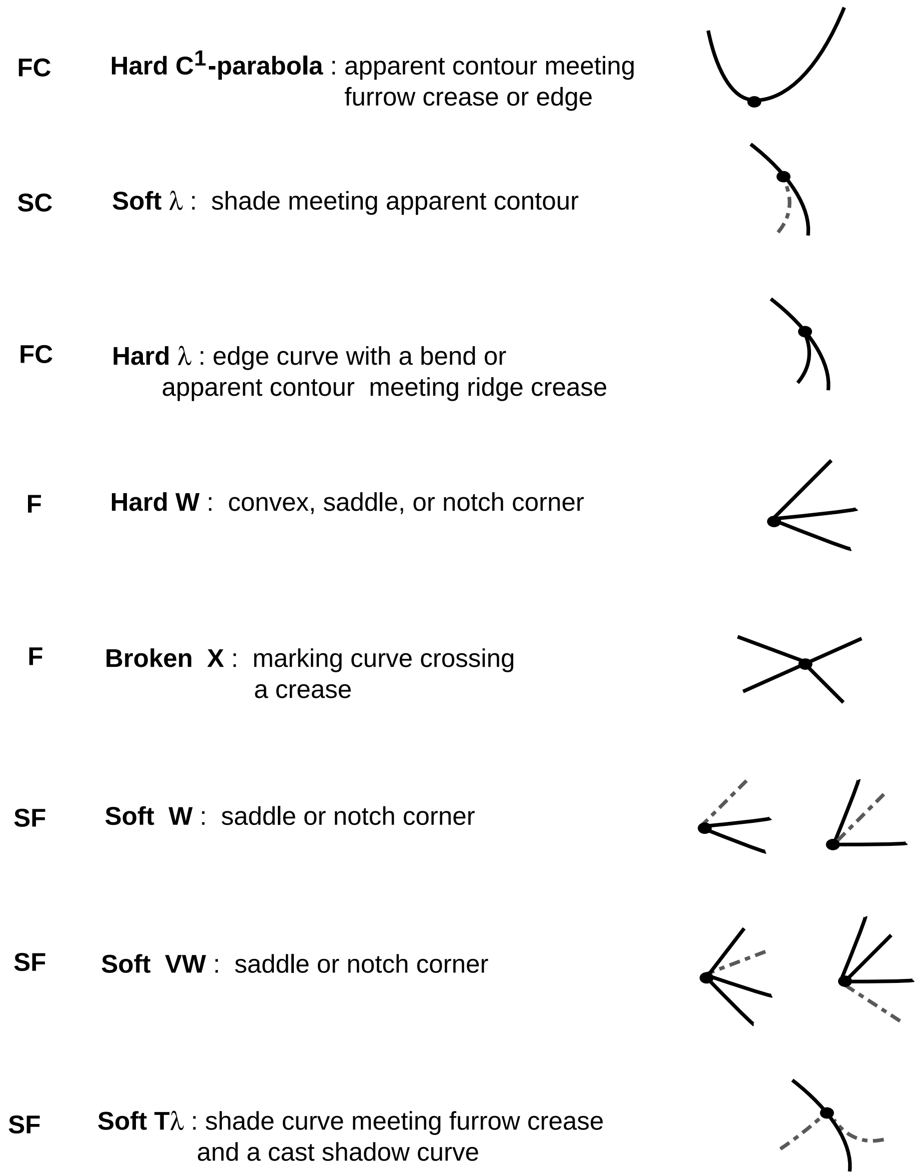


Figure 19. Second part of the "alphabet" of possible configurations of hard curves (solid black) and soft curves (dashed grey) corresponding to the local configurations of features-shade/shadow-apparent contours resulting from the classification.

5. Classification of Generic Transitions in Configurations

We give in this section the general form of the classification of generic transitions (5.1). This is further expanded in the more detailed classifications for (FC) given in (5.6) and (SC) given in (5.4). The detailed classifications for (SFC) and the multilocal cases are postponed until Part II [DGH2].

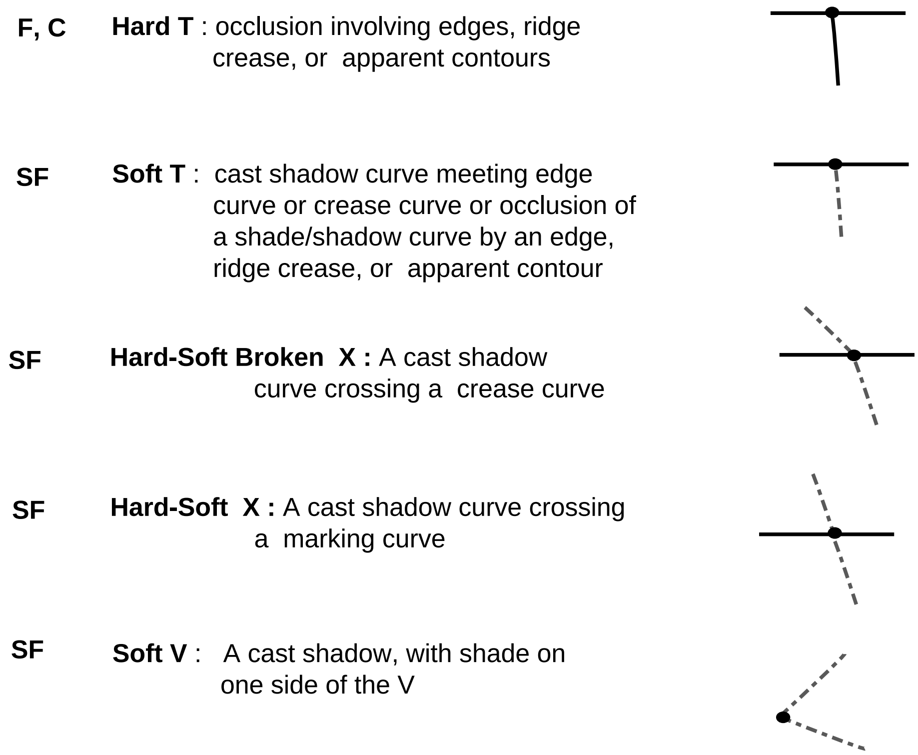


FIGURE 20. The "alphabet" of possible multilocal configurations of hard curves (solid black) and soft curves (dashed grey) - involving either occlusion of one object by another (involving edge, ridge crease, or apparent contours;or a cast shadow curve from a distant feature and an edge, marking curve, ridge crease, or apparent contour.

5.1 (**Generic Transitions for Configurations of Geometric Features, Shade or Shadow, and Apparent Contours).** *We consider the case of a single fixed light source and fixed objects in the scene and generic transitions involving geometric features, shade/shadow curves, and apparent contours. The generic transitions under viewer movement occur for both the local and multilocal configurations. First, the local generic transitions are given as follows.*

(1) *(C) The possible generic transitions for apparent contours on a smooth surface with uniform light/shade are given by Table 2. We do not consider this well-known case further in this paper. See* [Ko].
(2) *(SC) The possible generic transitions for configurations of shade/shadow curves with apparent contours are given by Table 4 and are explained in (5.4) .*
(3) *(FC) This classification is deduced from the abstract classifications of mappings to the plane preserving one of: marking curve, edge curve, crease, or corner. Except for corner transitions, the classification is explained in (5.6). The classification of generic transitions for corners will be given in Part II* [DGH2].

$\mathcal{A}$ Classes (smooth surface)	${}_X\mathcal{A}$ Classes (marking curve)	${}_X\mathcal{A}_e$ –codim	Edge Sings (surface with edge)	Crease Sings on one sheet of crease	Fold Shade Curve
reg. map	reg. image		reg. edge	reg. crease	reg.shade
fold	semifold	0	Y	Y	Y
"	semi lips/beaks	1	Y	Y	Y
"	semi-goose	2	Y	Y	NV
cusp	semi-cusp	1	Y	Y	Y
swallowtail	semi-swallowtail	2	Y	Y	Y
lips/beaks	lips/beaks on boundary	3	TV	TV	N
fold	boundary cusp	1	Y	Y	Y
fold	boundary rhamphoid cusp	2	Y	Y	Y
cusp	double cusp	3	TV	TV	N

TABLE 3. *Transitions of view projections for smooth surfaces with geometric features:* Abstract transitions for mappings preserving a smooth curve X, denoted "${}_X\mathcal{A}$–equivalence", yield common transitions for view projections on marking curves, boundary edges, crease curves, and fold shade curves (of the same type across a horizontal row). These yield the families of codimension 1 transitions in (4.1) with different geometric interpretations depending on the cases as illustated in Figures 26, 21, 22, 23, 24, and 25. Here Y indicates that both the singularity is realized by a projection of a surface with appropriate feature and moreover its versal unfolding is realized by viewer movement; N indicates the singularity cannot be realized; NV indicates the singularity can be realized but the versal unfolding cannot be; and TV indicates that the topologically versal unfolding can be realized by viewer movement (see §2).

(4) *(SFC) This classification is a refinement of that for (FC) taking into account the additional configuration structure resulting from the classification of stable interaction of shade/shadow curves with geometric features. The classification will be explained in detail in Part II* [DGH2].

Likewise, the multilocal generic transitions will be given in Part II [DGH2].

Remark 5.2. Just as with the classifcation for stable local configurations, there are also consequences of (5.1) for detecting features in Images by identifying implied relations between different local configurations. We expain these consequences in §7.

The complete classification includes both codimension 1 and 2 transitions. We first identify the remaining codimension 1 transitions before giving the complete classifications for the types of configurations.

5.3 (**Additional Codimension 1 Transitions for (SC) and (FC)**). *In addition to the three types of transitions given by Table 3, there are the following codimension* 1 *transitions for (SC) and (FC) which do not involve corner transitions.*

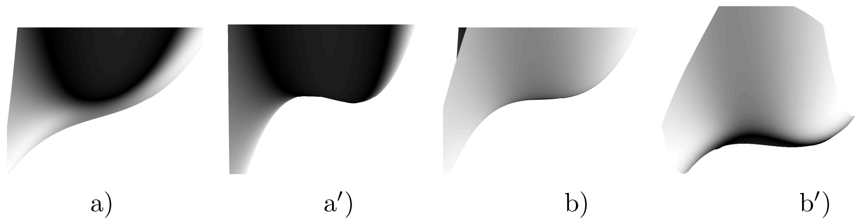


FIGURE 21. **Semi-Lips and Semi-beaks Transitions (SC) involving Apparent Contours and Shade Curves.** Lighting is from the right. a), a′) shows a semi-beaks transition where the shade curve (and the band of light) is broken by the apparent contour. b), b′) shows a semi-lips transition (same as a), a′) but viewed from the opposite direction). A sliver of shade appears as the view changes. These transitions also occur with the roles of light and shade interchanged.

4 Light Direction Cusp–Fold View (SC) (see Table 4): *This occurs for a cusp point of the light projection map. The contour generator for the fold apparent contour meets the shade/shadow curve transversally at the* C^1 *point. The transition moves the contour generator off of the* C^1 *point. See figure 27.*

5 Nontransverse Semi–Fold (FC) (type iii) in (2) of (5.6)) : *Five visually different transitions can occur when apparent contours from* both *sheets of a crease pass through the same point in the image, having a* C^1*-tangency. They occur for a surface with crease only when the two tangent planes to the smooth sheets coincide for one point along the crease. The surface sheets at that point are* non-transverse *and a valley crease turns into a ridge crease. The transitions occur as the semifold points on the two sheets come together and cross over at the non-transverse point on the crease. These are illustrated in figure 28.*

6 Fold apparent contour passing over isolated stable geometric feature point (FC) (see (3) of (5.6)) : *An isolated geometric feature point is a stable (F) type which differs from the other stable (F)-types within a neighborhood of it. The transition occurs when a fold contour generator curve passes in a generic way through the isolated point, such as a marking curve intersecting a crease. This is illustrated in figure 29.*

We now provide the more detailed descriptions for each of the classes of generic transitions. First, we begin with (SC).

5.4 (**Generic Transitions for Shade/Shadow - Apparent Contours (SC)**). *The generic transitions for local configurations involving shade/shadow curves and apparent contours are given in Table 4 which can be understood as follows:*

(1) *The possible transitions are modeled by viewer movement for the surfaces given in Table 4 with initial view from the positive* x*-axis; The explicit transitions of codimension* 1 *are shown in figures 21, 23 and 26.*

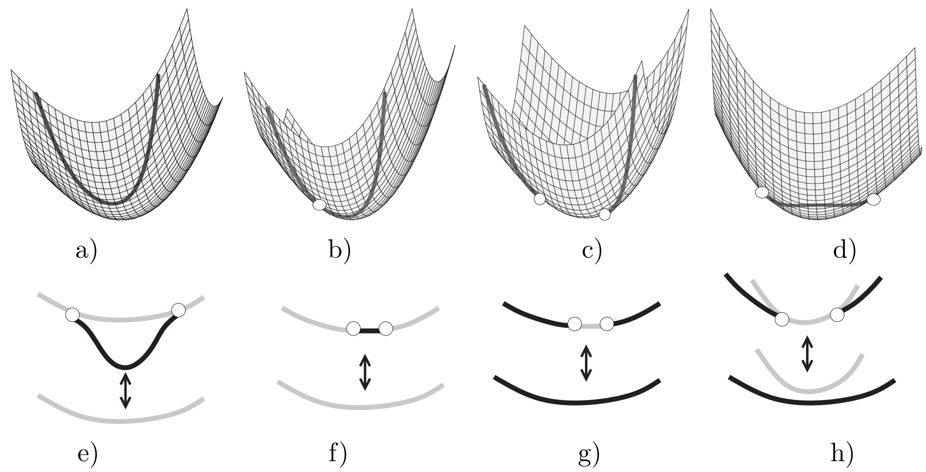


FIGURE 22. **Semi-lips and Semi-beaks Transitions (FC)**. Top row: Semi-beaks transition for marking curve (grey curve) a) to b) to c), where in b) the marking curve is tangent to contour generator *on the surface*: The view from the opposite side gives a semi-lips transition ending in d). Bottom row: schematic representations of semi-lips e), f), g) and semi-beaks h) transitions for boundary edges and creases (grey curves) and apparent contours (black curves). To visualize these, remove one of the halves of the surfaces in a) - d) along the marking curve, which then becomes either a boundary edge or crease curve (for one sheet). If the front half-surface is removed, a) to b) to c) corresponds to h) (moving upward), while from the opposite view we obtain g) (moving upward). If the back half-surface is removed, we obtain f), respectively e), (moving upward). For valley creases only cases e) and g) can occur. In case e) on a ridge crease, the section of apparent contour can be hidden by the other sheet.

(2) Fold Shade Curve: *The view projection maps correspond to the local mappings from surfaces with marking curve; but certain cases are not realized, as explained in Table 3.*
(3) Cusp Map for the Light Direction: *The combined shade/shadow curve defines a C^1–parabola. There are two cases for the view projection maps. See figure 27.*
(4) *With the one exception of the "semigoose", viewer movement gives the versal unfoldings of the local mappings in Table 4.*

Remark 5.5. The classification given by (5.4) and in table 4 corrects in a number of respects the classification given by Donati [Di] for codimension ≤ 2. We explain these differences in [DGH].

Next, we consider generic transitions in (FC).

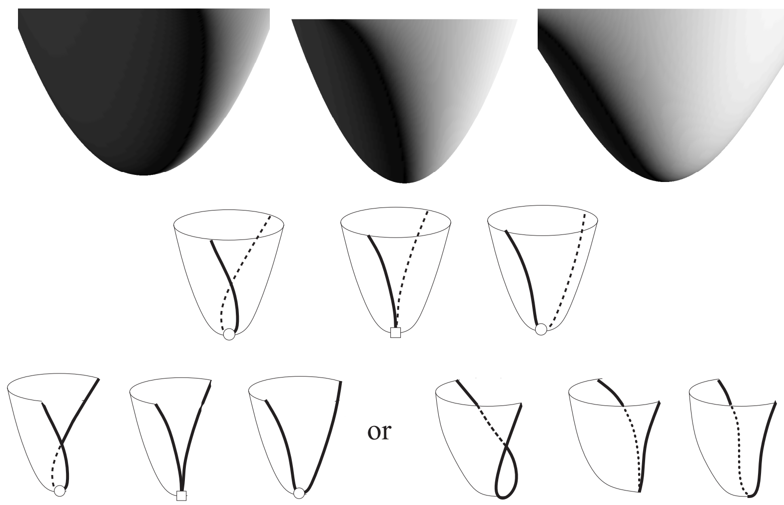


FIGURE 23. **Boundary Cusp Transition**. Top row: for a **shaded surface**(SC). Middle row: schematic transition for **marking curve** (FC) (or the shaded surface (SC) above). Occluded curves are indicated by dashes. Bottom row: for a **boundary edge** (FC); the two cases are views from opposite directions. The transition changes a (soft or hard) λ junction into a C^1-parabola.

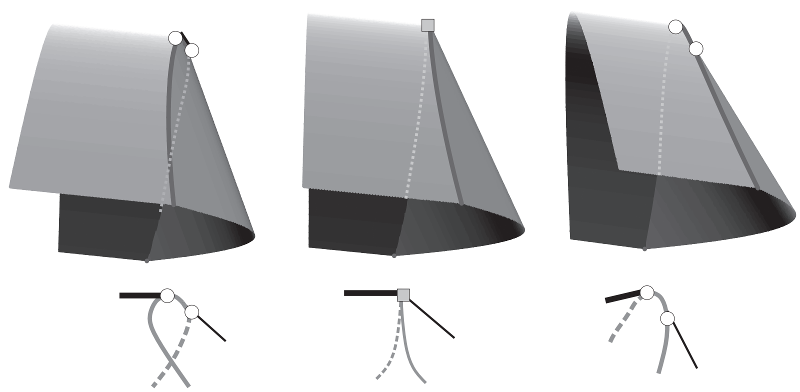


FIGURE 24. **A Boundary Cusp on a Crease Transition (FC)** Top row: One such transition is shown, with the corresponding schematic representation of the transition shown on bottom row, with crease edges (grey curves) and apparent contours on the two sheets (thick and thin black curves), with occluded curves (dashed) and visible T-junctions and λ-junctions indicated.

5.6 (**Generic Transitions for Geometric Features - Apparent Contours (FC)**). *The generic transitions for local configurations involving geometric features and apparent contours are given as follows:*

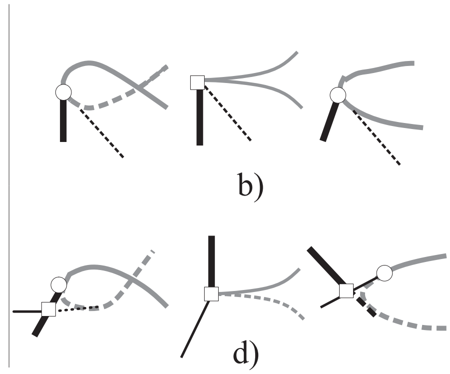


FIGURE 25. **Boundary Cusp on a Crease Transitions (FC)** Schematic representations of the transitions involving crease edges (grey curves) and apparent contours on the two sheets (thick and thin black curves), with occluded curves (dashed) and visible T-junctions and λ-junctions indicated. Cases a) (full transition shown in Fig. 25), b) and c) are ridge creases and d) is a valley crease. Only for a valley crease can T-junctions occur away from the cusp.

Surface $z = f(x,y)$	Name	$\mathcal{S}_e$-codim	Comments
$x^2 + xy + y^2$	semi-fold	0	
$(x + \varepsilon_1 y)^2 + \varepsilon_2 y^3$	semi-lips/beaks	1	$\varepsilon_1, \varepsilon_2 = \pm 1$
$x^2 + y^2 + x^2y + xy^2$	boundary cusp	1	
$xy + x^3 + y^2$	semi-cusp	1	
$xy + x^2 + y^3$	cusp for light direction, fold for view direction	1	See (5.4: 3)
$(x + y)^2 + y^4$	semi-goose	2	See (5.4: 4)
$x^2 + y^2 + xy^2 + x^4y$	boundary rhamphoid cusp	2	
$xy + y^2 + x^4 + \varepsilon x^6$	semi-swallowtail	2	$\varepsilon = \pm 1$ (*)
$xy + x^3 + y^3$	cusp maps for light and view directions	2	See (5.4: 3)

TABLE 4. (SC) Classification up to codimension 2 of view projections and generic transitions for interactions of shade/shadow and apparent contours. Surfaces given by the equations provide models for each type of interaction beginning with initial view along the positive x-axis. (*) For just qualitative properties of interactions, there is no difference between the two cases of the semi-swallowtail

(1) Edge Curve or Marking Curve on Smooth Surface: *The possible transitions are given by the columns in Table 3 and illustrated in figures 22, 23 and 26.*
(2) Crease Curves: *There are three possibilities: i) there are only apparent contours from one sheet; ii) the view projection of the crease has a singularity; and iii) there are apparent contours on sheets on each side of the crease.*

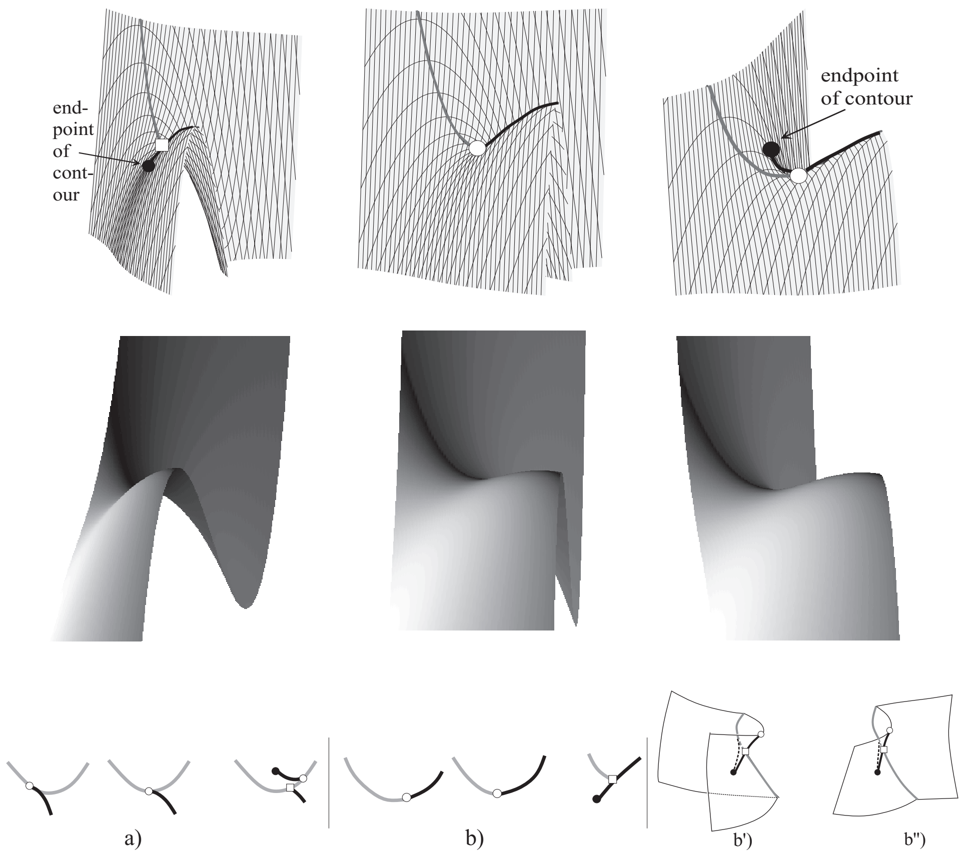


FIGURE 26. **Semi-cusp Transition** Top row: for (FC) **marking curve** (grey) and an apparent contour (black). A curve end and a (hard) T-junction change to a curve end and a (hard) λ-junction. Middle row: for (SC) **shaded surface**, with the shade curve replacing the marking curve. The junctions are now soft, and in the right-hand figure, the curve end is barely visible in the region lit only by background light. Bottom row: for (FC) **boundary edge** (grey curve) and an apparent contour (black) shown schematically. There are two cases: boundary edge a) entirely visible or b) only partially visible. For **(one sheet of) a surface with a crease** in place of boundary edge (FC): a) can occur only for a ridge crease, with the part above or below the grey edge occluded. (both surface sheets are locally visible); and b) can occur for a ridge crease with locally one surface visible, or for a valley crease with both visible. These are shown in b$'$) and b$''$) respectively, with the view direction moved to reveal the T-junction as in the right-hand figure of b).

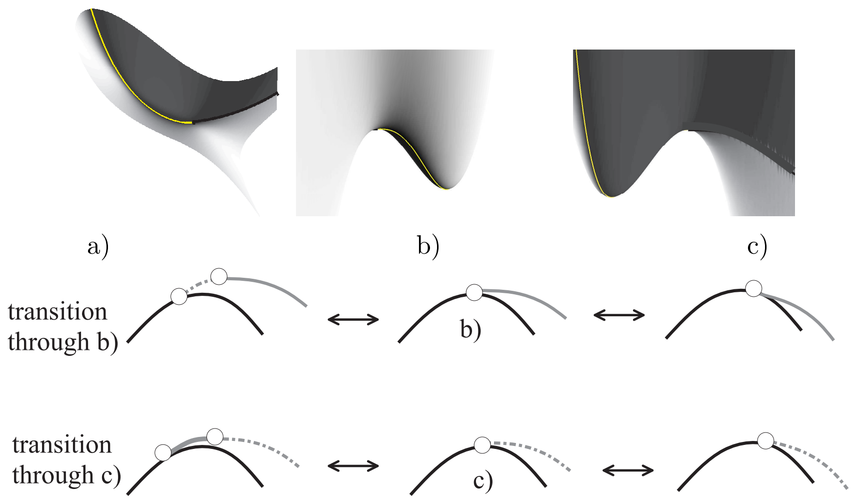


Figure 27. **Transition for cusp map in light direction - fold view direction (SC)** (see Table 4). Top row: a) stable view with apparent contour not touching the shade curve. The cast shadow curve, towards the right, and shade curve, towards the left (emphasized by white curve) together form a 'C^1-parabola' on the surface. In b) and c), the surface is the same but the view map is now a fold map: this is the 'light direction cusp - view fold' transition of **5.3 (5.3)**, with the same surface viewed from opposite directions. The transition point is in the center of each figure. The cast shadow line is only visible in c), and the shade curve is only visible near the transition point in b). The lower two rows are schematic representations of the transitions in b) and c) for the apparent contour (black), shade curve (solid grey), and cast shadow curve (dashed grey) as the viewpoint moves.

The cases for i) and ii) are given in column 5 *of Table 3 and illustrated in figures 22, 24, 25, and 26.*

For iii), if we allow creases where the two surfaces have the same tangent plane at a single point, then there are cases with apparent contours coming from each sheet do occur and are given in figure 28.

(3) Marking Curve Meeting Edge Curve or Crease: *The geometric configurations are given in c) - i) in figure 17. The transitions are those for an edge curve or crease, with an additional transition (and added codimension) for each case because of the movement of the transition point relative to the meeting point, as illustrated in figure 29.*
(4) Corners: *There are quite a few transitions for corners, taking into account each of the four types of corners (given in figure 6). We postpone giving the complete classification until part II of the paper.*

Remark 5.7. Cases 2i) and 2ii) were classified by Tari [Ta1], [Ta2]. We refine this classification taking into account visibility. For iii), these cases were excluded by Tari because they are "degenerate creases".

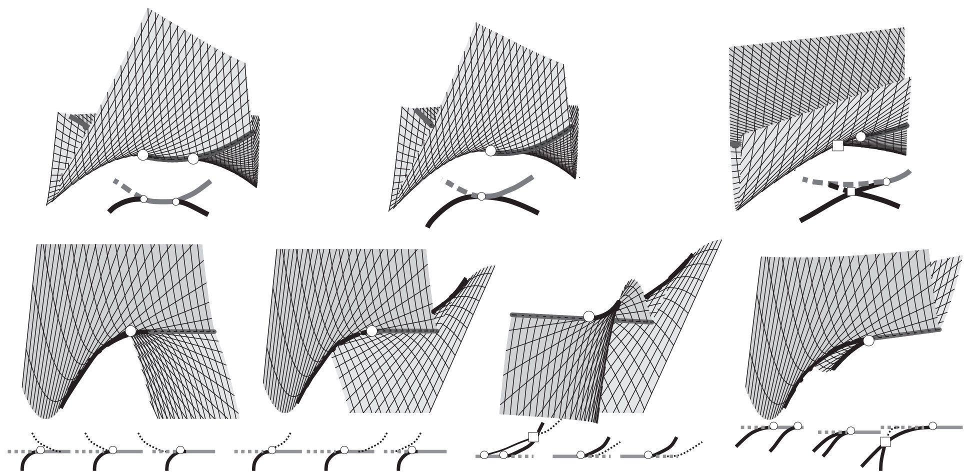


Figure 28. **Non-transverse Semi-fold Transition (FC)**: Five visually distinct transitions can occur on a surface with a crease,where at one point the tangent planes to the sheets coincide so that the crease changes from valley to ridge (see **5.3 (5.3)**). Top line: A transition where apparent contours of the two sheets come together and crossover (with the schematic depiction below the figures involving crease (grey curve) and apparent contours (black curves), and occluded curves (dashed)). The view from the opposite direction is qualitatively the same. Bottom line: the other four transitions, with only the figures for the transition point shown, but with the schematic depictions of the transitions below the figures.

6. Geometric Criteria for Singularities for View or Light

In this section, we briefly explain how the geometry of a surface affects the classifications which involve shade/shadow curves and apparent contours in the case of a smooth surface M without boundary edge, crease or marking curve. For the details see [DGH].

An apparent contour in the image corresponds on the surface M to a contour generator $\Sigma_{\mathbf{V}}$ which is the set of surface points $\mathbf{p}$ at which the view direction $\mathbf{V}$ lies in the tangent plane to M at $\mathbf{p}$. Similarly, a shade curve $\Sigma_{\mathbf{L}}$ on M, separating a region illuminated by the principal light source from one lit only by background illumination, is the set of points $\mathbf{p}$ at which the light direction $\mathbf{L}$ lies in the tangent plane to M at $\mathbf{p}$. Unlike a surface marking or crease or boundary edge, curves such as $\Sigma_{\mathbf{V}}$ and $\Sigma_{\mathbf{L}}$ are not arbitrary curves on M. The key property they possess has been observed by Koenderink [Ko, pp. 230, 243]. Let $\mathbf{U}$ be any direction in space, for example $\mathbf{U} = \mathbf{V}$ or $\mathbf{U} = \mathbf{L}$, and let $\Sigma_{\mathbf{U}}$ be the set of points on M at which the tangent plane contains $\mathbf{U}$.

6.1 (Characterization of Points on Contour Generators or Shade Curves)**.**
For a smooth surface M in $\mathbb{R}^3$ and a direction $\mathbf{U}$ in the tangent plane at $\mathbf{p}$, the tangent direction to $\Sigma_{\mathbf{U}}$ on M at $\mathbf{p}$ is conjugate to $\mathbf{U}$.

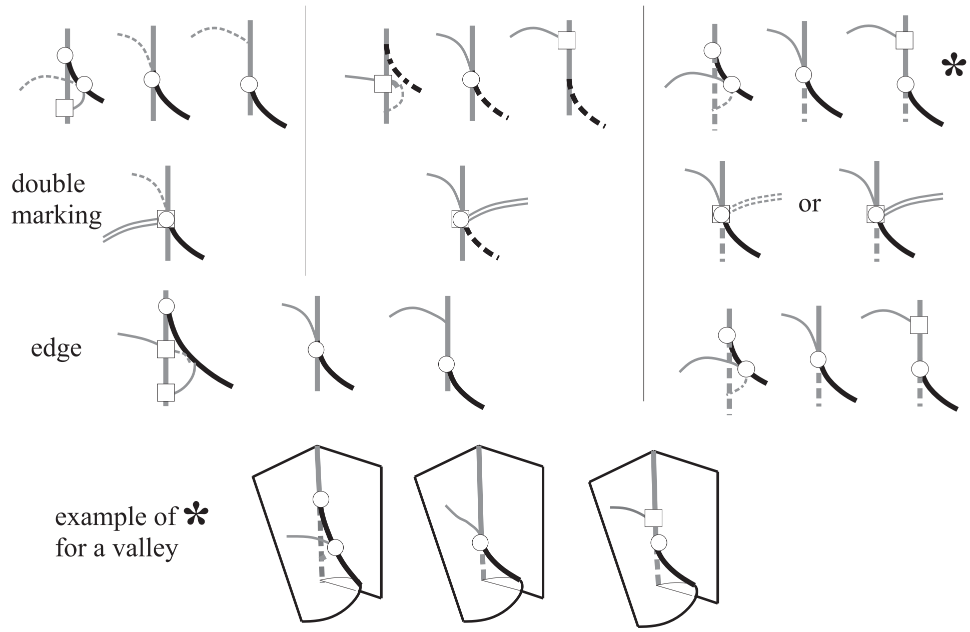


FIGURE 29. **Transition for apparent contour and meeting of a marking curve and crease/edge (FC)**: , see **5.3 (29)** and compare with the stable cases in Fig. 17). Top row: marking curve (thin grey curve) on only *one* sheet of the crease surface, with crease curve (thick grey line), with apparent contour (black curves) on the sheet (occluded curves are dashed and occluded junctions are not marked). There are three possible cases (in the three columns) depending on visibility. The first two can occur only for a ridge crease but the third can occur for either a ridge or a valley. Second row: case of marking curve on each crease sheet (marking curve on second sheet is double grey curve) shown at transitional point. In the third column, the ridge is the left-hand figure and the valley is the right-hand figure. Third row: case of marking curve meeting an edge. There are only two possibilities, and the second one is identical with the corresponding crease case. Bottom row: transition on a valley crease corresponding to the schematic diagram indicated by a *.

Here, 'conjugate' is relative to the second fundamental form of the surface at **p**. The properties of conjugacy which we need here are as follows.

(i) In an elliptic (convex or concave) region of M, every tangent direction $\mathbf{U}$ has a unique conjugate, different from $\mathbf{U}$, and distinct tangent directions have distinct conjugates.

(ii) At every point in a hyperbolic (saddle-shaped) region there are two special tangent directions called asymptotic directions, where the tangent line pierces the surface; each of these two is conjugate to itself, but distinct tangent directions still have distinct conjugates.

(iii) At a point of the parabolic curve separating these regions, there is exactly one asymptotic direction and all other tangent vectors are conjugate to this one. Thus, by Property (6.1), all directions $\mathbf{U}$ apart from the asymptotic direction have corresponding curves $\Sigma_{\mathbf{U}}$ tangent to the unique asymptotic direction. (When $\mathbf{U}$ is the asymptotic direction $\Sigma_{\mathbf{U}}$ becomes singular, either an isolated point or a crossing of two branches.)
(iv) From (i) and (ii) it follows that, if two distinct tangent vectors at $\mathbf{p}$ have the same conjugate direction, then $\mathbf{p}$ is a parabolic point.

As an immediate consequence note that assuming, as we always do, $\mathbf{V}$ and $\mathbf{L}$ to be distinct, it follows that the curves $\Sigma_{\mathbf{L}}$ and $\Sigma_{\mathbf{V}}$ are tangent at $\mathbf{p}$ if and only if $\mathbf{p}$ is parabolic. Certain singularities require $\Sigma_{\mathbf{L}}$ and $\Sigma_{\mathbf{V}}$ to be non-tangent (transverse) and others require them to be tangent (see [BG2]). This gives (i) and (ii) of the following geometric constraints on interactions involving apparent contours and shade curves and other similar considerations lead to the remaining parts (compare Table 3).

6.2 (Geometric Constraints on Apparent Contours and Shade Curves)**.**
(i) semi-fold, semi-cusp, semi-swallowtail, boundary cusp and boundary rhamphoid cusp can never *occur at a parabolic point;*
(ii) semi-beaks and semi-goose can only *occur at a parabolic point, in fact, the semi-goose requires that* $\mathbf{p}$ *be a "cusp of Gauss", that is a point where the asymptotic direction is tangent to the parabolic curve;*
(iii) lips or beaks on the boundary and double cusp can never *occur;*
(iv) viewer movement cannot "versally unfold" the semi-goose in the sense that there are known local singularities close to a semigoose (in fact, semi-cusp singularities) which viewer movement cannot achieve.

For example, here is the argument leading to (iii) in the case of a double cusp, which means that the contour generator and the shade curve both have a cusp in the image. If this can occur then the view direction $\mathbf{V}$ must be tangent to both of these curves on the surface at $\mathbf{p}$. Since they are tangent, and $\mathbf{V} \neq \mathbf{L}$, we deduce that the point $\mathbf{p}$ is parabolic, and that $\mathbf{V}$ is the asymptotic direction at $\mathbf{p}$. But viewing a parabolic point along an asymptotic direction does not give a cusp; rather it gives a "lips/beaks singularity" in the image since $\Sigma_{\mathbf{V}}$ is an isolated point or a crossing (see for example [Ko, pp. 303, 458]). This is a contradiction.

7. Comments and Summary

We have explained in this paper how the complex interactions of geometric features, light, and viewer movement can be analyzed using the methods of singularity theory to yield a classification of both expected local features of images and their generic transitions under viewer movement. These provide a concise alphabet of local curve configurations that we expect to see in images, along with the possible geometric properties that accompany them. As well we provide a specific classification of the generic transitions which occur in these configurations under viewer movement. We have explicitly provided the generic transitions for interaction of shade/shadows and apparent contours (SC) and geometric features with apparent contours (FC) with the exception of the list of corner transitions. These along with the classification of generic transitions for interactions of all three (SFC), and for multi-local interactions will be presented in Part II. Together these results provide a catalogue which subsumes and considerably refines the earlier work of a number of authors on special aspects of images.

There are two important consequence of the classification for detecting 3D properties of objects in images.

First, in contrast with the use of the traditional T-junction as an indicator of occlusion, not all T-junctions indicate occlusion. Moreover, there are several other cases apparently similar to T–junctions, but which are characterized instead by the tangency of a curve segment to another curve (this configuration more closely resembles the Greek letter λ than a T). These λ–junctions occur as three different types, with several representing a number of different cases.

Second, there are cases where two different curve segments meet tangentially, but only agree to first order. So they can be locally modeled as C^1–versions of parabolas, a typical example being two half parabolas, say $y = x^2, x \leq 0$ and $y = 2x^2, x \geq 0$, meeting and having the same tangent line at the origin but with a discontinuity in the curvatures as we pass this C^1–point.

These two characterizing features suggest that to detect geometric properties of objects we must include the actual behavior of tangent lines to our various types of curves to measure either tangency or sudden change in curvature as we approach the special points to distinguish among the types in the alphabet.

There are additional consequences for detecting features in Images which result from the knowledge of the generic transitions given in 5.1 even in the case of a single fixed image. In particular, for the perturbed view on each side of the generic transitions, there are typically several local stable features, which cannot normally be related to each other. However, the transitions suggest that certain combinations of the features indicate the presence of a relation which then helps identify more easily 3D features of objects in images. These are "higher order relations" not a consequence of purely local or even multilocal stable configuration information.

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