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1 **Running head: Stochastic reef dynamics**

2 **Title: Stochastic dynamics of a warmer Great Barrier Reef**

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15

16 **Abstract:** Pressure on natural communities from human activities continues to increase. Even
17 unique ecosystems like the Great Barrier Reef (GBR), that until recently were considered near-
18 pristine and well-protected, are showing signs of rapid degradation. We collated recent (1996-
19 2006) spatio-temporal relationships between benthic community composition on the GBR and
20 environmental variables (ocean temperature and local threats resulting from human activity). We
21 built multivariate models of the effects of these variables on short-term dynamics, and developed
22 an analytical approach to study their long-term consequences. We used this approach to study the
23 effects of ocean warming under different levels of local threat. Observed short-term changes in
24 benthic community structure (e.g., declining coral cover) were associated with ocean temperature
25 (warming) and local threats. Our model projected that in the long term there was a very high
26 likelihood of low ($\leq 10\%$) coral cover. With increasing temperature and/or local threats, corals
27 were initially replaced by sponges, gorgonians, and other taxa, with an eventual moderately high
28 probability of domination ($> 50\%$) by macroalgae when temperature increase was greatest (e.g.,
29 3.5°C of warming). Our approach to modeling community dynamics, based on multivariate
30 statistical models, enabled us to project how environmental change (and thus local and
31 international policy decisions) will influence the future state of coral reefs. The same approach
32 could be applied to other systems for which time series of ecological and environmental
33 variables are available.

34

35 **Keywords:** Great Barrier Reef, coral reef, reef state, communities, dynamics, compositional data,
36 ocean temperature, local threat, stochastic model, long-term behavior, climate change, human
37 impacts.

38

39 **Introduction**

40
41 Natural communities are under threat from human perturbation and the effects of climate change
42 (Halpern et al. 2008, Butchart et al. 2010). Despite clear evidence of degradation in many habitat
43 types (Duffy 2003, Worm et al. 2006), the size and direction of long-term impacts remain
44 uncertain, because few ecological monitoring programs are older than a few decades. Coral-reef
45 communities are one of the clearest examples of a biological system greatly altered by human
46 activities, including overfishing, increased nutrient loading, and anthropogenic warming (Hughes
47 et al. 2003). Globally, coral cover has declined to 10-20%, and corals have been replaced to
48 some degree by other invertebrates such as gorgonian soft corals and sponges, by crustose
49 coralline algae, algal microturfs and bare carbonate substrate (collectively termed CTB: Aronson
50 and Precht 2000), and by fleshy macroalgae (Aronson et al. 2002, Bruno and Selig 2007, Bruno
51 et al. 2009, Schutte et al. 2010). This broad decline of coral cover has led to a general flattening
52 or simplification of reef habitats with direct consequences for fishes and other reef inhabitants
53 (Alvarez-Filip et al. 2009).

54
55 Ocean warming has been a primary cause of mass coral mortality and coral cover decline over
56 the last two to three decades (Hughes et al. 2003, Hoegh-Guldberg and Bruno 2010, Selig et al.
57 2012). Temperatures $\sim 1^\circ\text{C}$ greater than the local seasonal maximum can disrupt the relationship
58 between corals and their symbiotic zooxanthellae, leading to “coral bleaching” (Baker et al.
59 2008). In some circumstances, bleaching can cause partial or complete mortality of coral
60 colonies. Mortality and mass bleaching have been observed across the Pacific and Indian
61 Oceans, and the Caribbean (Glynn 1991, Baker et al. 2008, Eakin et al. 2010). Anomalously high
62 water temperature is also associated with coral disease outbreaks (Bruno et al. 2007, Harvell et

63 al. 2009, Rogers and Muller 2012), possibly due to an increase in susceptibility of the coral host
64 caused by thermal stress and bleaching (Mydlarz et al. 2009).

65 Although the proximate causes of coral population declines (e.g., disease, bleaching, and
66 pollution) have been identified, relatively little progress has been made in deciphering the
67 relative importance of different drivers. Thus, our understanding of how these drivers affect
68 entire reef communities (not just coral cover) is incomplete. Moreover, little progress has been
69 made on using the large empirical record of reef degradation to develop analytical models of
70 future reef composition. By linking changes in community structure with changes in
71 environmental conditions, we should be able to identify key environmental drivers. These data
72 can also be used to move beyond the usual univariate studies of reef health (e.g. De'ath et al.
73 2012) into multivariate studies of community dynamics.

74
75 The purpose of this study was to project the composition of future coral reef benthic
76 communities under current environmental conditions, and under environmental change
77 scenarios. We used data from the Great Barrier Reef to build multivariate models for the effects
78 of ocean temperature and “local threat level” (an index of local human impacts developed for the
79 Reefs at Risk Revisited report, Burke et al. 2011) on short-term changes in reef composition. We
80 then used these simple empirical models and a novel analytical approach to project the long-term
81 distributions of reef composition under both current environmental conditions and increased
82 ocean temperature, and local threat level. We also estimated the probability of undesirable reef
83 compositions, in which coral cover is reduced to $\leq 10\%$ or when macroalgae dominates $> 50\%$ of
84 the benthos.

85

86 **Methods**

87 *Data*

88 Data from Australia's Great Barrier Reef (GBR) were obtained from quantitative reef surveys.
89 Video transect surveys of 46 reefs (locations: Appendix, Fig. A1) were performed over at least
90 two consecutive years between 1996 to 2006 as part of the Australian Institute of Marine Science
91 long-term monitoring programme. The methods are described in Abdo et al. (2004) and
92 summarized in Appendix A.1. Reef data consisted of proportional benthic cover of three
93 biological categories: coral, macroalgae and other (which includes CTB, sponges, gorgonians,
94 and other invertebrates). The data we use were aggregated to reef level, and are a subset of the
95 data in Bruno et al. (2009) and Żychaluk et al. (2012). These data formed multivariate time series
96 of reef composition in consecutive years (62 series, median length 7 years, length range 2 to 11
97 years). We analyzed the combined data as 364 pairs of observations in consecutive years.

98
99 For each reef, we extracted data on sea surface temperature (SST) climatology (the long-term
100 value for a 4 x 4 km square, as defined in Selig et al. 2010), annual mean anomalies (departure
101 from long-term value for this 4 x 4 km square), and local threat level as described in Appendix
102 A.1. We used one-year lags for both climatology and anomaly, and centred and scaled them to
103 mean zero, standard deviation 1. The Reefs at Risk Revisited local threat level index (Burke et
104 al. 2011) is a categorical variable (with 4 levels: 1) low, 2) medium, 3) high, and 4) very high)
105 that summarizes information on coastal development, marine-based pollution and damage,
106 watershed-based pollution, and overfishing, most of which was resolved to the 1 km or 3 km
107 scale (Burke et al. 2011). It is important to note that we have little information about the effects
108 of high and very high local threat, because we had only one reef (with 5 and 9 pairs of

109 observations in consecutive years) for each of these two categories. We also considered distance
110 from the coast as another potential proxy for human activity, but this was strongly related to
111 local threat index (Appendix, Fig. A2), and models using distance from the coast always
112 performed worse than corresponding models using local threat index (Appendix, Table A1).

113 *Short-term change in reef composition*

114 We represent the reef compositions on a single reef in two consecutive years by the column
115 vectors $\mathbf{y}(t)$ and $\mathbf{y}(t + 1)$. Each such vector has three components $y_1(t)$, $y_2(t)$, $y_3(t)$,
116 representing the proportions of coral, algae, and other at time t , and summing to 1. We described
117 short-term changes in composition (from one year to the next) using perturbing vectors
118 (Appendix A.2), which are themselves compositions. If there is no change in composition
119 between two years, the corresponding perturbing vector is $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. For each element of the
120 perturbing vector, a value greater than $1/3$ indicates an increase in that component, and a value
121 less than $1/3$ indicates a decrease.

122 *Model assumptions*

123 We assume that the perturbing vectors on each reef are independent of those on other reefs, that
124 future perturbing vectors are conditionally independent of past reef composition given current
125 reef composition, that the process generating these perturbing vectors is homogeneous over time
126 (conditional on the values of environmental variables), and that measurement error is small
127 relative to the short-term variability in the true composition of a reef. We argued in Żychaluk et
128 al. (2012), supporting information, section S1.2, that similar assumptions will often be
129 approximately true, and that models based on them are useful descriptions of the regional
130 dynamics of coral reefs.

131 *Models for short-term change*

132 For a single species, a linear model for changes in log abundance between successive time points
133 is the natural starting point for an investigation of the factors affecting population dynamics,
134 because exponential growth results in a straight-line relationship between log abundance and
135 time. In the same way, a linear model for isometric log-ratio (ilr) transformed perturbing vectors
136 (Appendix A.3) is a natural starting point, because exponential growth of all components results
137 in a straight-line trajectory in ilr coordinates (Egozcue et al. 2003). We do not expect that all
138 components will grow exponentially, so we include the effects of current reef composition in our
139 model, which takes the form

$$140 \text{ilr } \mathbf{p}(t) = \mathbf{c} + \mathbf{A}\mathbf{x}(t) + \beta_1 z_1(t) + \boldsymbol{\epsilon}(t). \quad (1)$$

141 Each term in Equation 1 is a column vector with two elements. The response variable ilr $\mathbf{p}(t)$ is
142 the transformed short-term change in reef composition. The first term on the right of Equation 1
143 (\mathbf{c}) is a constant for any given reef and environmental change scenario, which depends on
144 climatology and local threat. The second term ($\mathbf{A}\mathbf{x}(t)$) is the effects of current transformed reef
145 composition. The third term is the effect of SST anomalies. The fourth term ($\boldsymbol{\epsilon}(t)$) describes the
146 stochastic effects of processes such as storms, diseases, and crown of thorns starfish, for which
147 we do not have data (and for which we assume mean vector zero and constant covariance
148 matrix). More detail on Equation 1 is given in Appendix A.4. All the parameters in Equation 1
149 can be back-transformed to compositions and represented on ternary plots, in the same way as
150 the perturbing vectors. We fitted and checked this model, tested hypotheses, and visualized
151 parameters as described in Appendices A5-A7.

152

153 Our model is the multivariate equivalent of the widely-used stochastic Gompertz model. The
154 univariate version is a plausible description of the density-dependent dynamics in many single-

155 species time series (e.g. Dennis et al. 2006), and the multivariate version is likewise a good way
156 to approximate the dynamics of a multi-species community (Ives et al. 2003, Hampton et al.
157 2013). Independently, Gross and Edmunds (in review) arrived at a very similar model for reef
158 dynamics.

159 *Long-term behaviour and effects of changes in sea surface temperature*

160 Under the simplifying assumption that annual mean SST anomaly is a sequence of identically
161 normally distributed random variables, independent of past SST anomalies and of the error term
162 $\epsilon(t)$, the model in Equation 1 may converge to a stationary distribution, which can be found
163 analytically (Appendix A.8). This stationary distribution tells us about the long-term behavior of
164 the GBR under current conditions. We then used two approaches to explore the effects of
165 changes in the long-term mean u_2 of climatology on long-term behaviour: sensitivity to
166 infinitesimal changes and calculation of stationary distributions under a range of long-term
167 means. We think that changing climatology rather than changing anomalies is the right way to
168 model the effects of long-term change in SST, because the climatology parameter describes the
169 long-term mean temperature at a site. However, we comment in the Discussion on the
170 consequences of this assumption. We assumed that the variance of SST anomalies did not
171 change, which greatly simplifies the sensitivity analysis. The evidence for changes in the
172 temporal variability of recent and projected temperatures remains ambivalent (Huntingford et al.
173 2013), so it would be difficult to justify any other treatment.

174

175 It is possible to calculate the sensitivity of the stationary density at any point to changes in
176 climatology (Appendix A.9). The contour of zero sensitivity is of particular interest because it
177 separates reef compositions projected to become less likely under increased climatology (those

178 with negative sensitivity) from reef compositions projected to become more likely under
179 increased climatology (those with positive sensitivity). A similar approach can be used to express
180 the long-term effects of local threat level in terms of equivalent increases in climatology
181 (Appendix A.9). Although local threat effects and climatology effects do not necessarily have the
182 same direction, the component of a local threat effect that acts in the same direction as the
183 climatology effect tells us how much the difference between two local threat levels is worth in
184 terms of climatology.

185
186 We also examined the effects of changes in climatology on the stationary distribution of reef
187 composition using numerical methods. We calculated stationary distributions for a range of
188 climatologies between the current regional minimum (rounded down to the nearest degree) and a
189 value 3.5°C warmer than the current regional mean. These climatologies cover a plausible range
190 of future ocean temperatures. Increases of 0.83 to 3.91°C in global mean surface temperature by
191 2100 compared to 2000 are projected under the four Representative Concentration Pathways
192 (Meehl et al. 2012). Under a range of climate models, sites in the GBR may experience 0.76 to
193 1.01°C increase in maximum summer SST per °C increase in global mean temperature
194 (Wooldridge et al. 2012). Thus, an increase of several °C in climatology seems plausible, despite
195 the large uncertainty. We caution that examining plausible future climatology involves
196 extrapolating beyond the range of currently-observed climatology. In contrast, the sensitivity
197 calculation outlined in the previous paragraph looks at the effects of small increases in
198 climatology, and does not require extrapolation.

199 *Probability of undesirable compositions*

200 To summarize the changes in stationary distributions across a range of climatologies, we report
201 the probabilities of low coral cover (the stationary probability that coral cover is less than or
202 equal to 10%) and high algal cover (the stationary probability that algal cover is greater than
203 50%). The 10% low coral cover threshold is believed to be the minimum cover required for net
204 reef accretion (Kennedy et al. 2013), whereas the 50% high algal cover threshold is a
205 conventional definition of macroalgal dominance (Bruno et al. 2009). These statistics can be
206 interpreted in two ways: as the long-run proportion of time we expect the composition of an
207 individual reef to satisfy the specified condition; and as the proportion of randomly-chosen reefs
208 we expect to satisfy the specified condition, at a given point in time.

209 **Results**

210 *Short-term change in reef composition*

211 The most obvious pattern in the raw data (Fig. 1A) was that most reefs had low algal cover most
212 of the time, with occasional but generally short-lived excursions towards higher algal cover.

213 There was a wide range of coral cover. Perturbing vectors, which represent short-term changes in
214 composition (Fig. 1B), were clustered around the coral-other 0.5-isoproportion line, covering its
215 whole length. Thus, large increases and decreases in algae occurred, but in general the ratio of
216 coral to ‘other’ changed little in the short term. Large decreases in macroalgal cover tended to be
217 associated with unusually cold SST anomalies (Fig. 1B, blue symbols predominate in left half of
218 plot). It was not easy to discern a difference in short-term changes between local threat
219 categories (Fig. 1B, different symbol shapes).

220 *Fitted model*

221 Current composition, SST anomaly and climatology, and local threat had significant effects on
222 transformed perturbing vectors (Appendix, Tables A2 and A3). If the ratio of algae to coral was

223 high, the proportion of algae tended to decrease the following year, with little effect on the ratio
224 of coral to ‘other’ (Fig. 1C, light blue dot (2)). Conversely, if the ratio of ‘other’ to the geometric
225 mean of coral and algae was high, the proportion of ‘other’ tended to decrease the following
226 year, and the ratio of algae to coral tended to increase (Fig. 1C, dark blue dot (3)). A one
227 standard deviation increase in SST anomaly tended to increase the proportion of algae, with
228 little effect on the relative proportions of coral and ‘other’ (Fig. 1C, green dot (4)). A one
229 standard deviation increase in climatology had an effect in the same direction as the SST
230 anomaly effect, but with a slightly smaller magnitude (Fig. 1C, pink dot (5)). Post-hoc tests
231 (Appendix, Table A3) showed that only the medium local threat level was significantly different
232 from the low local threat level. Relative to low local threat, reefs in the medium local threat
233 category tended to have short-term changes that decreased the ratios of coral to both algae and
234 ‘other’ (Fig. 1C, yellow dot (6)). In subsequent results, we therefore looked separately at the low
235 and medium local threat levels. The lack of evidence for effects of the high and very high local
236 threat levels (Fig. 1C, orange (7) and red (8) dots respectively) may be due to the small number
237 of observations in these categories (5 and 9 pairs respectively, and in each case from a single
238 reef). Thus, although the high threat category appears to be associated with decreases rather than
239 increases in algal cover (Fig. 1C, orange dot (7)), the confidence ellipse for this effect overlaps
240 both the no-effect point, and the confidence ellipses for the effects of medium and very high
241 threat.

242

243 No major departures from the model assumptions were apparent. We checked by simulation that
244 our parameter estimates were qualitatively robust to plausible levels of observation error
245 (Appendix A.6, Fig. A3). However, observation error may lead to underestimation of the effects

246 of increased climatology (Appendix, Fig. A4). Removing 30 out of 364 pairs of observations that
247 were identified as outliers (Appendix A.7, Fig. A5) did not substantially affect parameter
248 estimates (Appendix A.7, Fig. A6). It was noticeable that in observations with large increases in
249 algae, the model under-predicted these increases (Appendix A.7, Fig. A7B). Although this
250 involves relatively few observations, it may be biologically important. There were no strong
251 patterns in residuals plotted against explanatory variables (Appendix, Fig. A8), or time
252 (Appendix, Fig. A9), and residuals were not strongly spatially autocorrelated (Appendix, Fig.
253 A10).

254 *Long-term behaviour under current environmental conditions*

255 There was strong evidence for the existence of a stationary distribution (Appendix A.10). Under
256 current climatology, this distribution was unimodal for both low (Fig. 2A) and medium (Fig. 2B)
257 local threat levels. The level of uncertainty in the stationary distributions was fairly high,
258 especially for compositions with high stationary density (Appendix, Fig. A11), but the stationary
259 distributions of individual bootstrap replicates all had similar shapes.

260
261 In the long term, under current climatology and low local threat level, likely reef compositions
262 had high cover of 'other', moderate coral cover, and low algal cover (Fig 2A). For medium local
263 threat level (Fig. 2B), this distribution shifted towards compositions with lower coral cover and
264 higher 'other' and algal cover.

265 *Effects of changes in sea surface temperature and local threat level.*

266 The sensitivity of the stationary density to small changes in climatology provides an analytical
267 estimate of likely effects of long-term increases in sea surface temperature. At low local threat,
268 the zero contour representing no effect (Fig. 3A, black line) roughly divided compositions with

269 low algal cover, which became less likely (blue), from compositions with high algal cover,
270 which became more likely (red). The largest increases in stationary density (reddest) were for
271 compositions with low coral and algal cover and high cover of 'other'. For medium local threat
272 level (Fig. 3B), the zero contour moved toward the right, so that compositions with low coral
273 cover became more likely, and compositions with high coral cover less likely. The set of
274 compositions with the highest increases in stationary density (reddest) was moved towards
275 somewhat higher algal cover and lower coral cover than in the low local threat level, but the
276 relative cover by 'other' remained the largest component in this scenario. For both local threat
277 levels, the uncertainty associated with sensitivity was substantial (Appendix, Fig. A12). The
278 long-term effect of the difference between medium and low local threat levels was equivalent to
279 the effect of 2.8°C increase in climatology, but with high uncertainty (95% confidence interval
280 (1.1, 19.4)°C increase). Numerical results confirmed this pattern. With a 2°C increase in
281 climatology, the stationary distribution under low threat level (Fig. 2C) shifted away from high
282 coral cover, and towards high 'other' and somewhat higher algal cover, compared with current
283 conditions (and became more similar to the current distribution under medium local threat). At
284 medium local threat level, a 2°C increase in climatology caused a shift away from 'other' in the
285 direction of higher algal cover (Fig. 2D).

286
287 Animations (available online) show more information about the relationship between the
288 stationary distribution of reef composition and climatology. For low local threat level (Appendix
289 A11), as climatology increased, coral cover declined, leading to a state with both low coral cover
290 and low algal cover at around 1.5°C increase. At higher climatology, coral cover remained low
291 and algal cover increased. At around 3.25°C increase, the stationary distribution was bimodal,

292 with high density associated with low coral cover and either low algae and high ‘other’, or high
293 algae and low ‘other’. This bimodality arises because the stationary distribution has a large
294 enough spread that, for high climatology, the stationary mean is positioned so that large amounts
295 of density get squashed into both the ‘other’ and algae vertices. Thus, alternative stable states
296 may be possible under some future environmental conditions. For medium local threat level
297 (Appendix A12), coral cover was low for current climatology, and most of the probability was
298 associated with high cover of ‘other’. The distribution moved towards increased algal cover with
299 increases in climatology, but the stationary distribution did not appear bimodal.

300 *Probability of undesirable compositions*

301 The probability of low coral cover (Fig.4A and B) and high algal cover (Fig. 4C and D)
302 increased with climatology. However, the probability of low coral cover was greater than the
303 probability of high algal cover at any given climatology (this must be partly because the current
304 stationary distribution has most of its mass much further from the 50% algal threshold than from
305 the 10% coral threshold). Compared with the low local threat level, the probability of low coral
306 cover was greatly increased at medium local threat level (Fig. 4A vs. 4B), but there was less
307 change in the probability of high algal cover (Fig. 4C vs. 4D).

308 **Discussion**

309 *Observed and projected effects of ocean warming*

310 Our results highlighted differences between observed short-term and projected long-term
311 responses of reef composition to ocean warming. Over the period (1996-2006) covered by our
312 data, the observed short-term effect of increased ocean temperatures on reef composition was to
313 increase macroalgal cover, with proportional decreases in coral and ‘other’. However, moderate
314 future warming ($\sim 2^\circ\text{C}$) in our long-term projections led to dominance by ‘other’ (a category

315 including organisms such as sponges, gorgonians, and CTB), with algal dominance only
316 projected under extreme warming ($>2^{\circ}\text{C}$). Empirical evidence for phase shifts from coral to
317 ‘other’ states (Aronson et al. 2002, Norström et al. 2009), and for the relative rarity of
318 macroalgal dominance at the global scale (Bruno et al. 2009), is consistent with our analysis.
319 Thus, it may be more appropriate to think of macroalgae as fast-colonizing ephemeral taxa rather
320 than as competitive dominants under current conditions on the GBR (Connell 1987). However,
321 the potential for dynamics within the dominant ‘other’ category (Aronson et al. 2002) makes
322 resolving this category more finely a priority. The differences between the observed short-term
323 response to warming and our projected long-term dynamics occurred because short-term
324 increases in algae are modified in the long-term by reef composition in all successive years (Fig.
325 5, Appendix A.9). The result that short- and long-term effects of environmental change are in
326 different directions is a general one, and is likely to apply to almost all ecosystems (Appendix
327 A.9).

328
329 Although moderate warming moves the stationary mean towards dominance by ‘other’ rather
330 than by macroalgae, such warming also increases the proportions of reefs projected to have high
331 algal ($\geq 50\%$) and low coral ($\leq 10\%$) cover (Fig. 4). This is because the whole of the stationary
332 distribution is shifted clockwise, around the edge of the simplex, moving its tails away from the
333 coral vertex and towards the algal vertex (see animations: Appendices A11 and A12). These
334 proportions can be thought of in two ways. For a single reef, they are the proportions of time a
335 single reef spends at low coral, or high algal cover. For a population of reefs with the same
336 environmental conditions, they are the proportions of reefs with low coral and/or high algal
337 cover at a given time. The 10% threshold for coral cover is somewhat arbitrary, but is generally

338 believed to be the approximate minimum value required for net-reef accretion (Kennedy et al.
339 2013). Current coral cover on the GBR is only ~14%, down from 28% in the mid-1980s, and
340 even more so from a probable historical baseline of >50% (Hughes et al. 2011, Bruno 2013).
341 Our results suggest that warming of an additional 1-2°C will make further coral loss nearly
342 inevitable.

343
344 When studying the effects of increased temperature, we used the climatology parameter rather
345 than the anomaly parameter to model the effects of long-term warming. The estimated effect of
346 climatology on year-to-year changes includes the effects of spatial differences in species
347 composition and local adaptation, which may explain why the estimated climatology effect is
348 weaker than the estimated anomaly effect. We implicitly assume that changes in species
349 composition and opportunities for local adaptation can occur temporally, as well as spatially. If
350 this is not the case, then we will have underestimated the effects of long-term warming.
351 Nevertheless, because the directions of the climatology and anomaly parameters are very similar,
352 the model's direction for the long-term effect of warming is likely approximately correct.

353 *Local threats*

354 Being in the medium local threat category (compared with the low local threat category) had an
355 effect on short-term changes in composition roughly equivalent to 2.8 °C of warming.
356 Consequently, medium threat reefs are expected to have high levels of 'other' even under current
357 conditions, and low levels of coral and high levels of macroalgae are more likely than on low
358 threat reefs. The local threat metric encapsulates impacts from coastal development, marine-
359 based pollution and damage, watershed-based pollution, and overfishing (Burke et al. 2011). For
360 example, terrestrial run-off of sediment, nutrients, pesticides, etc. have a variety of negative

361 effects on corals, and can benefit sponges and seaweeds, effectively shifting community
362 composition away from corals, and towards ‘other’ and/or algae, as our model projected
363 (Fabricius 2005). Most of the study reefs were in the low local threat category, so there may be
364 little scope for further reduction in local threat. Furthermore, because ocean temperature
365 increases of 1-2 °C are likely (IPCC 2007), maintaining reefs in the low local threat category will
366 not alone be sufficient to secure the future of the GBR. Reducing both human perturbations and
367 the effects of climate change is necessary (Hoegh-Guldberg et al. 2007, Mumby and Steneck
368 2008, Sale 2008). Because Reefs at Risk Revisited is a static classification, we can say nothing
369 about how these threat categories might vary over time. Also, because the classification
370 integrates a wide variety of local threats, it would not be easy to design a management policy
371 based specifically around these threat categories.

372 *Complementary modelling approaches*

373 We have greatly expanded the scope of our previous work on statistical models of reef dynamics
374 (Żychaluk et al. 2012), and addressed the concern that these models ignored among-reef
375 heterogeneity in environmental conditions (Mumby et al. 2013). Conceptually, our approach (a
376 multivariate statistical model for reef dynamics) is closely related to statistical summaries of
377 empirical data on changes in coral cover (e.g. De'ath et al. 2012). However, using a multivariate
378 model reveals a difference in the direction of environmental change effects between the short and
379 long term, that would be undetectable using univariate analyses. Recently, simple analytical
380 models (e.g. Fung et al. 2011, Baskett et al. 2014) have advanced our understanding of how the
381 range of possible reef dynamics depends on biological features such as macroalgal growth rates
382 and coral life history characteristics. Our model is much less sophisticated as a description of
383 reef dynamics, although it can be viewed as a linear approximation of a more complicated

384 nonlinear dynamical system (Ives et al. 2003), and can answer some of the same questions about
385 dynamics. For example, Gross and Edmunds (in review), using a method very similar to ours,
386 showed that coral reefs from different habitats in the US Virgin Islands varied in their stability
387 properties in ways consistent with known features of coral life histories. Our model knows much
388 less biology than ambitious and sophisticated models of reef dynamics (e.g. Melbourne-Thomas
389 et al. 2011, Kennedy et al. 2013, Sebastian and McClanahan 2013). Unlike these models, we
390 cannot even attempt to predict what might happen to an individual reef. However, we can make
391 projections about the statistical properties of ensembles of reefs (analogous to “climate” rather
392 than to “weather”). We see these diverse modeling approaches as complementary. Given their
393 differences in assumptions, it may even be productive to use multimodel ensembles (Gardmark
394 et al. 2013) to look for robust projections about coral reef futures.

395
396 In summary, our models allowed us to explore regional community dynamics of the GBR. The
397 short- and long-term responses of the system to environmental change were quite different,
398 because of population-dynamic effects. This is likely to be true in many other systems. Statistical
399 models of community dynamics have the potential to bridge the gap between analytical theory
400 and field data, and have been found useful in systems including freshwater plankton (Ives et al.
401 2003, Hampton et al. 2013) and marine fisheries (Lindegren et al. 2009), as well as coral reefs
402 (Gross and Edmunds, in review).

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- 569 **Ecological Archives material**
570 Appendix A: additional methods and results
- 571 Appendix B: effects of climatology on stationary distribution, low local threat level.
- 572 Appendix C: effects of climatology on stationary distribution, medium local threat level.
- 573 Supplement (CooperRcode.zip): Zip file of R code used in analysis.

574
575

preprint

576 **Figures**

577 Figure 1. (A) Time series of Great Barrier Reef composition at 46 locations between 1996 and
578 2006. Each series of observations on the same reef in consecutive years is represented by a grey
579 line, starting at an open blue circle and ending at a filled orange circle. (B) Short-term changes in
580 reef composition for the data in (A), coloured by annual mean sea surface temperature anomaly
581 with a one-year lag. Circles: low local threat. Triangles: medium local threat. Diamonds: high
582 and very high local threat. White lines: 0.5-isoproportion lines, along which two of the
583 components of the composition have no change in relative proportions. For example, points
584 along the line from the algae vertex to the point bisecting the coral-‘other’ edge have no change
585 in the relative proportions of coral and ‘other’. (C) Parameters from Equation 1 in a model for
586 the data in (B). Each parameter is represented by its contribution to short-term change, with an
587 approximate 95% confidence ellipse. The intersection of the white lines corresponds to no effect.
588 Grey (1): intercept. Light blue (2) and dark blue (3): \mathbf{a}_1 and \mathbf{a}_2 columns of the matrix \mathbf{A} , which
589 describes effects of reef composition. Green (4): effect of centred and scaled SST anomaly. Pink
590 (5): effect of centred and scaled SST climatology. Yellow (6), orange (7), red (8): effects of
591 medium, high and very high relative to low local threat level, respectively. Grey dashed line:
592 shape of the covariance matrix Σ , represented by an ellipse at unit Mahalanobis distance around
593 the no-effect point.

594

595 Figure 2. Stationary distributions for the GBR at current climatology (A: low local threat, B:
596 medium local threat), and with a 2°C increase in climatology (C: low local threat, D: medium
597 local threat). Darker colours are more likely compositions.

598

599 Figure 3. Sensitivity of stationary density for the GBR to climatology, evaluated at current
600 climatology and either low (A) or medium (B) local threat. Blue: compositions that would
601 become less likely under small increases in climatology. Red: compositions that would become
602 more likely under small increases in climatology. Black line: compositions that would become
603 neither more nor less likely under small increases in climatology.

604

605 Figure 4. Probability of low coral cover (A and B: less than or equal to 10%) and high algal
606 cover (C and D: more than 50%) in the GBR over a range of climatology from the current
607 minimum (rounded down to the nearest degree) to 3.5°C warmer than the current mean. Solid
608 black lines: bootstrap mean probability. Dashed lines: 95% bootstrap confidence interval.
609 Vertical dotted line: current mean climatology. Horizontal grey bar: observed range of
610 climatology.

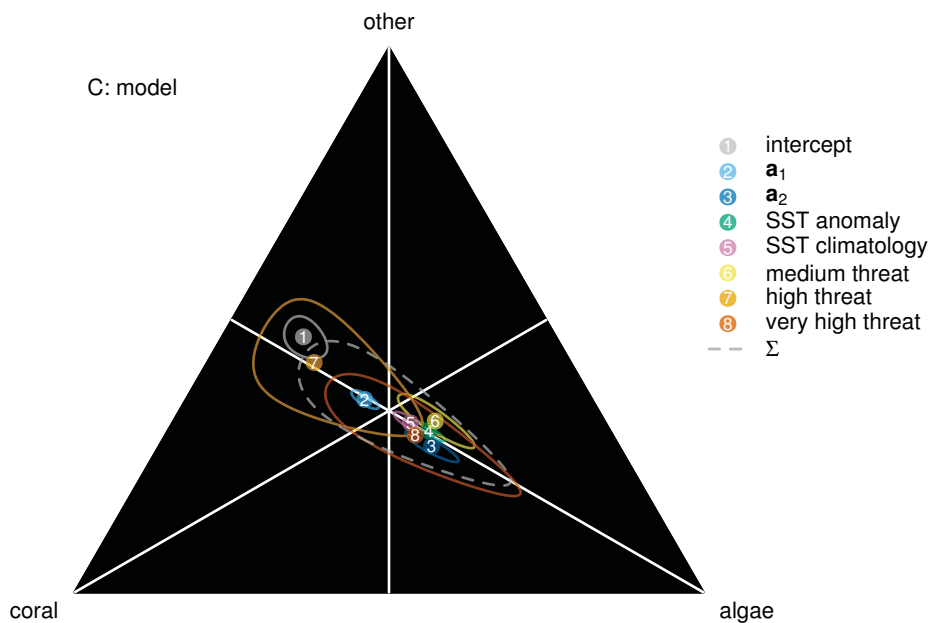
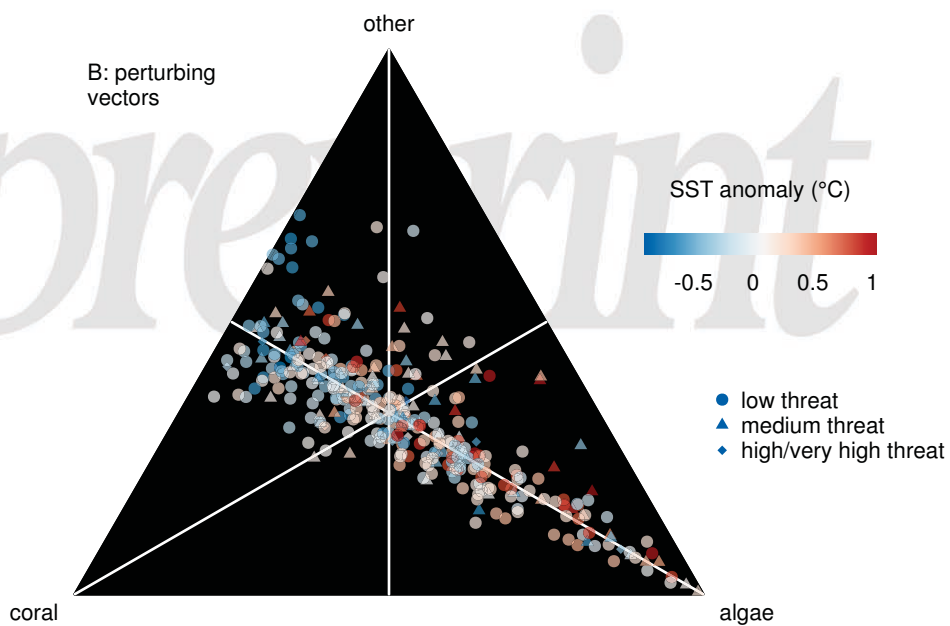
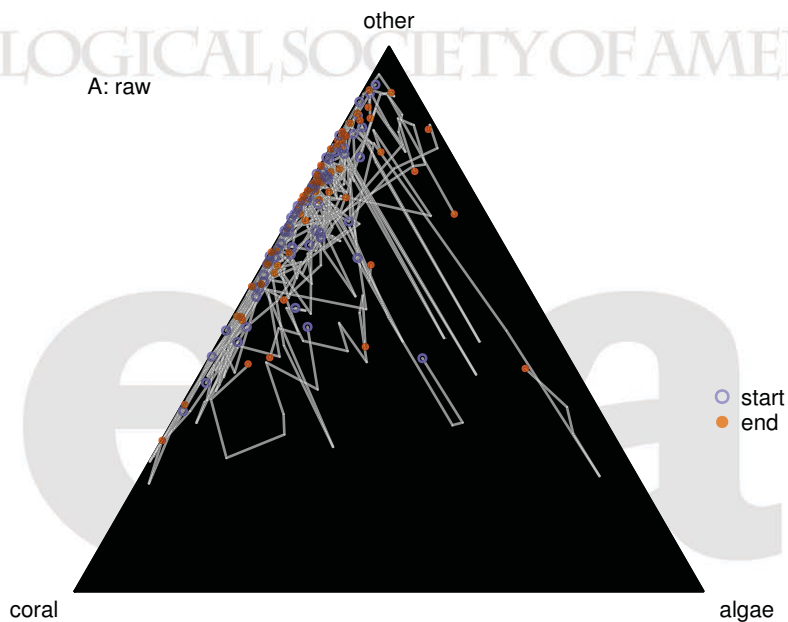
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612 Figure 5. Differences between short- and long-term effects of climatology on reef composition.
613 Solid arrow: direction of short-term effect of increased climatology on ilr-transformed perturbing
614 vector (tail of vector at the point representing a zero effect). Dashed arrow: direction of long-
615 term effect of increased climatology on stationary mean reef composition (tail of arrow at current
616 stationary mean, low local threat). The dashed arrow is a straight line in ilr coordinates. Both
617 arrows are scaled by an amount corresponding to a 3.5°C increase in climatology.

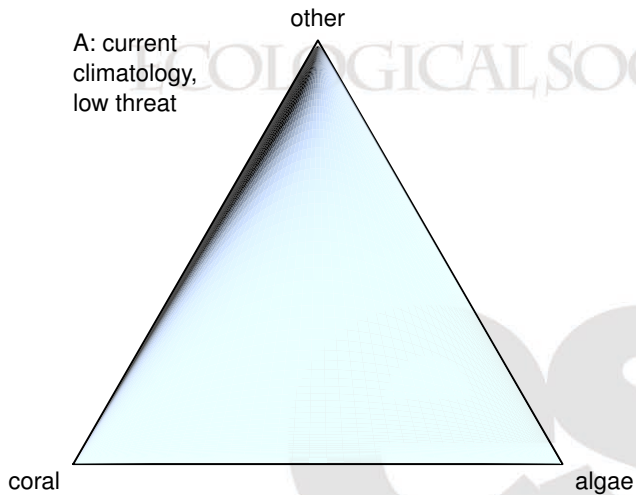
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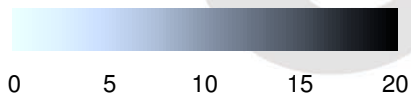
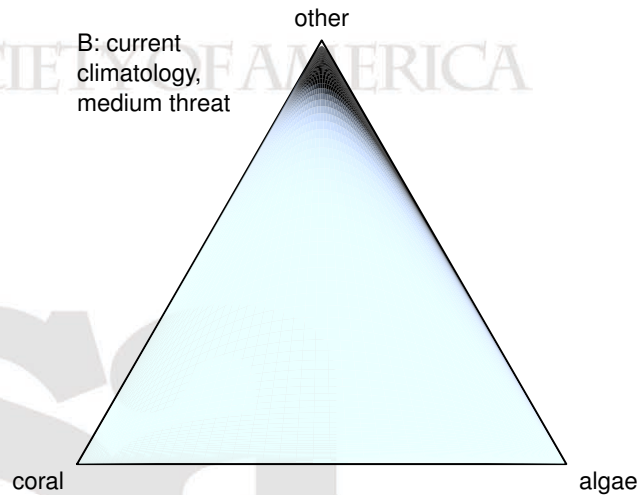
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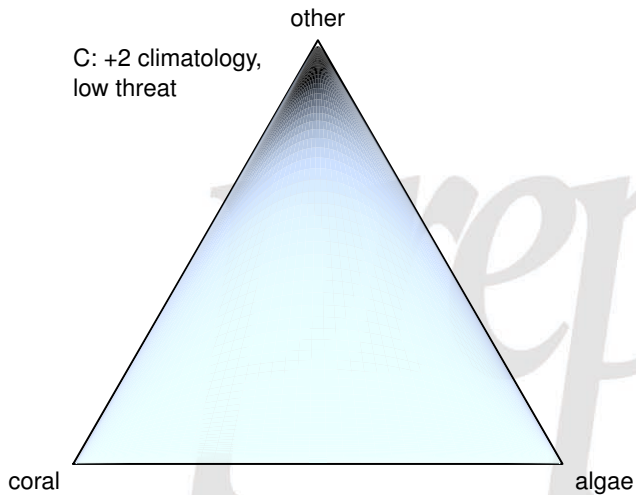
A: current
climatology,
low threat



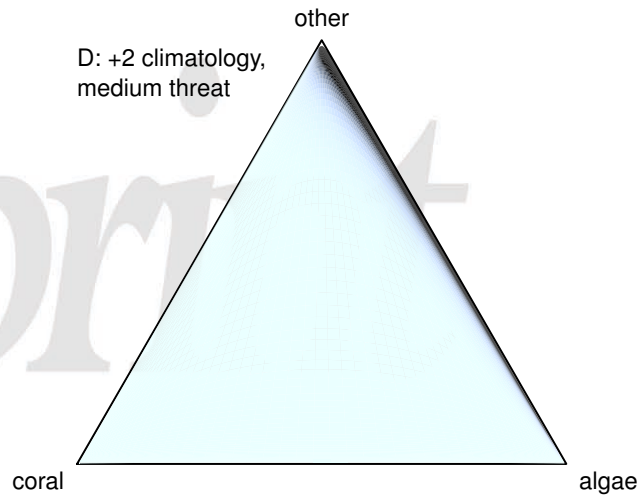
B: current
climatology,
medium threat



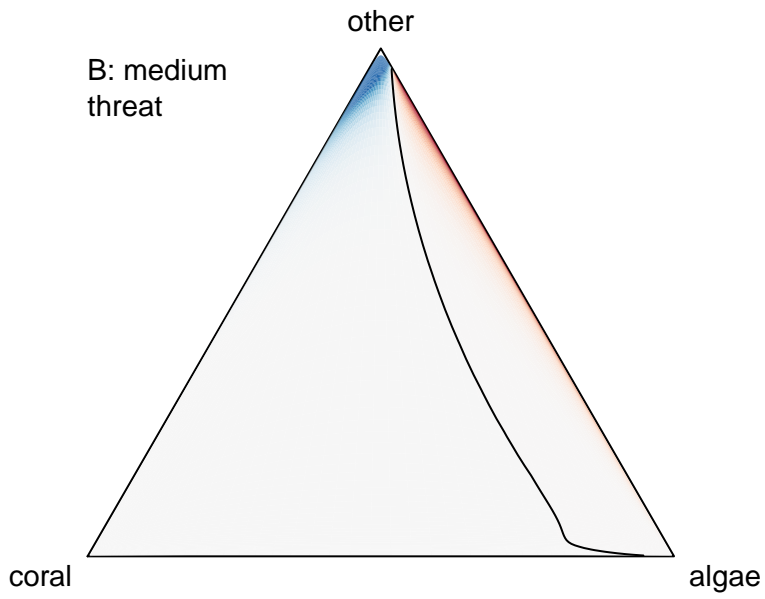
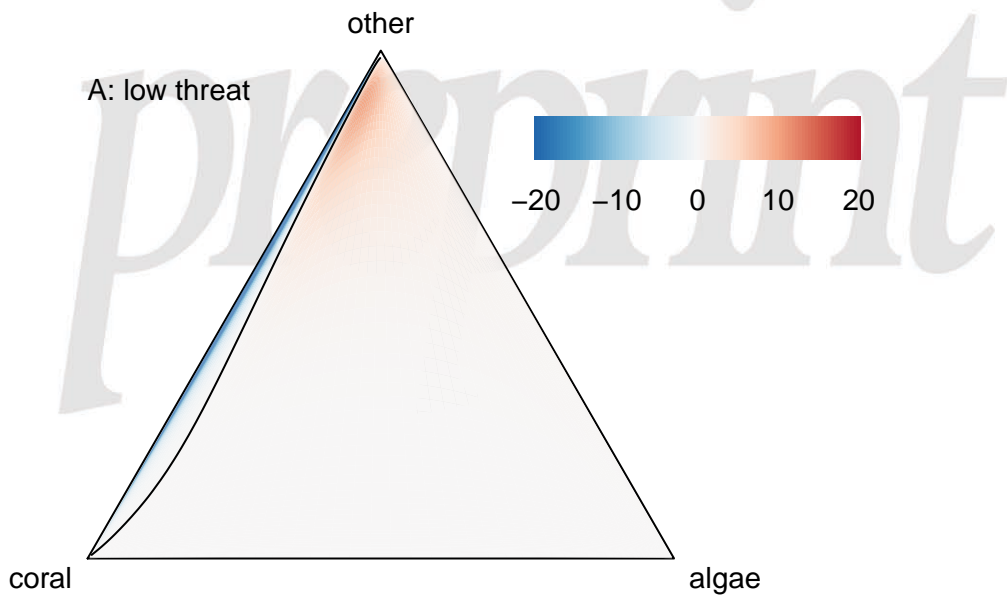
C: +2 climatology,
low threat

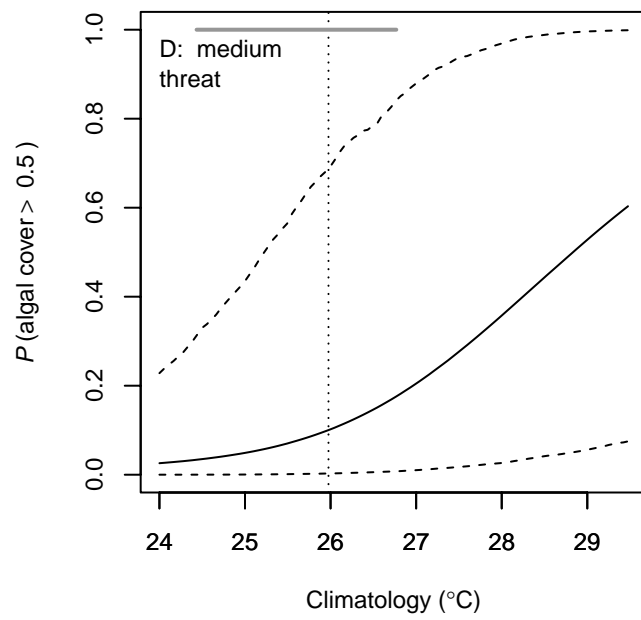
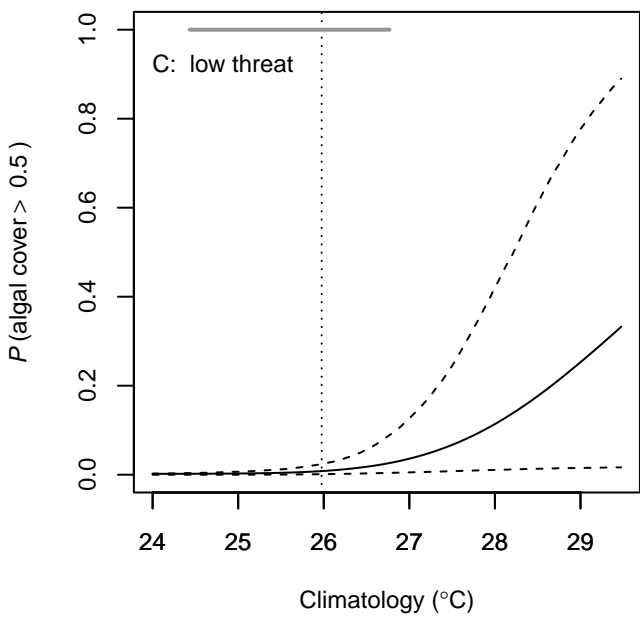
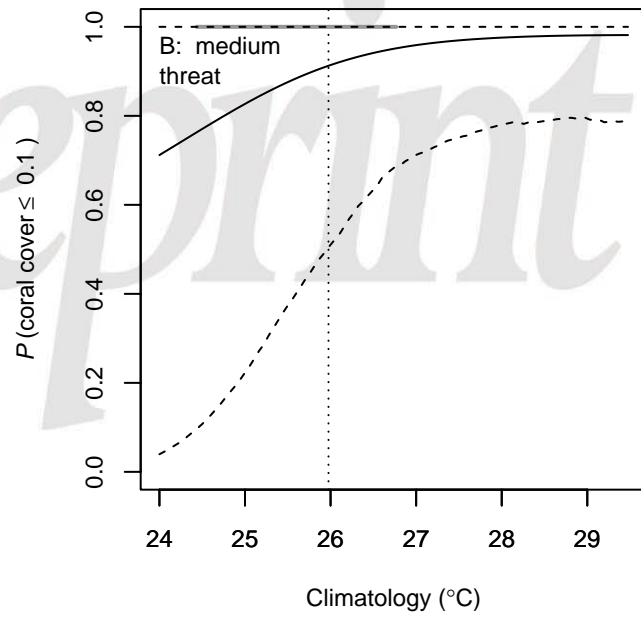
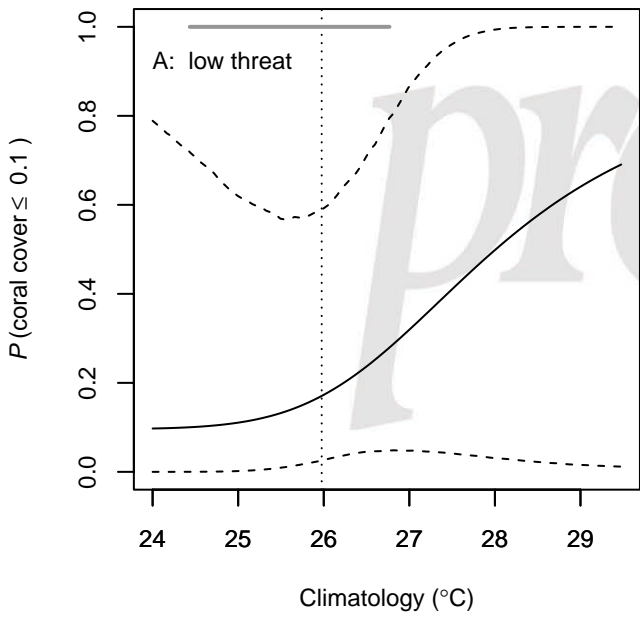


D: +2 climatology,
medium threat



esa





esa

— short-term: $\frac{d\mathbf{c}}{d u_c} = \frac{1}{s_c} \beta_2$

- - - long-term: $\frac{d\mu^*}{d u_c} = \frac{1}{s_c} (\mathbf{I} - \mathbf{B})^{-1} \beta_2$

